Generalized Parametric Path Problems

Uncertainty in Artificial Intelligence (UAI), 2021

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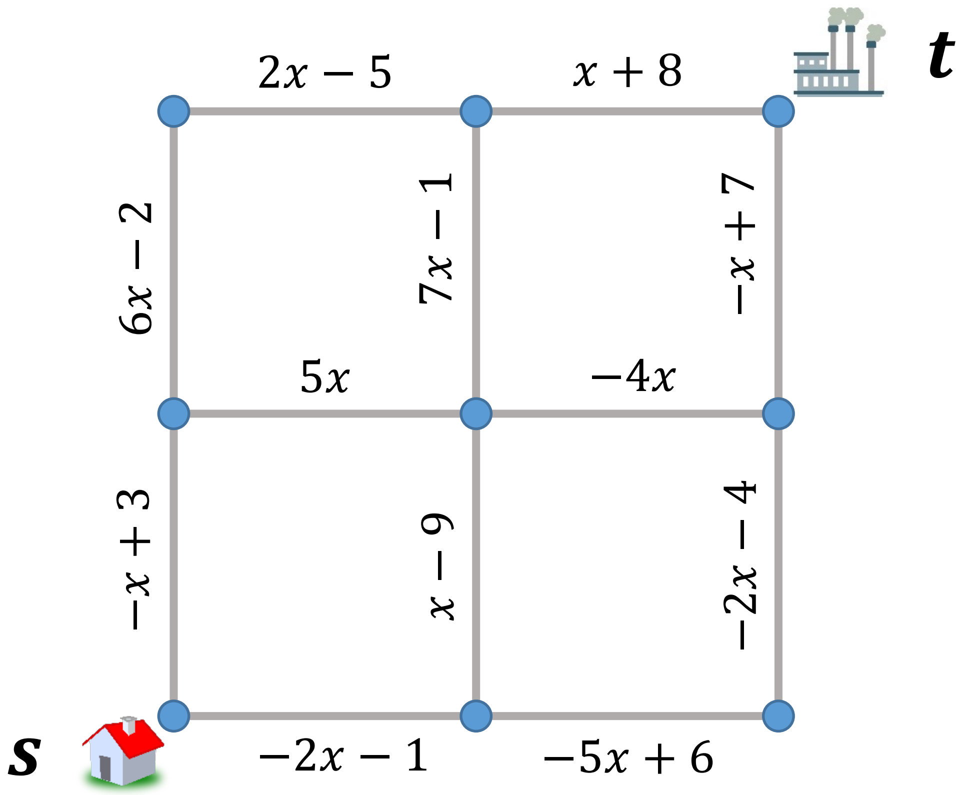
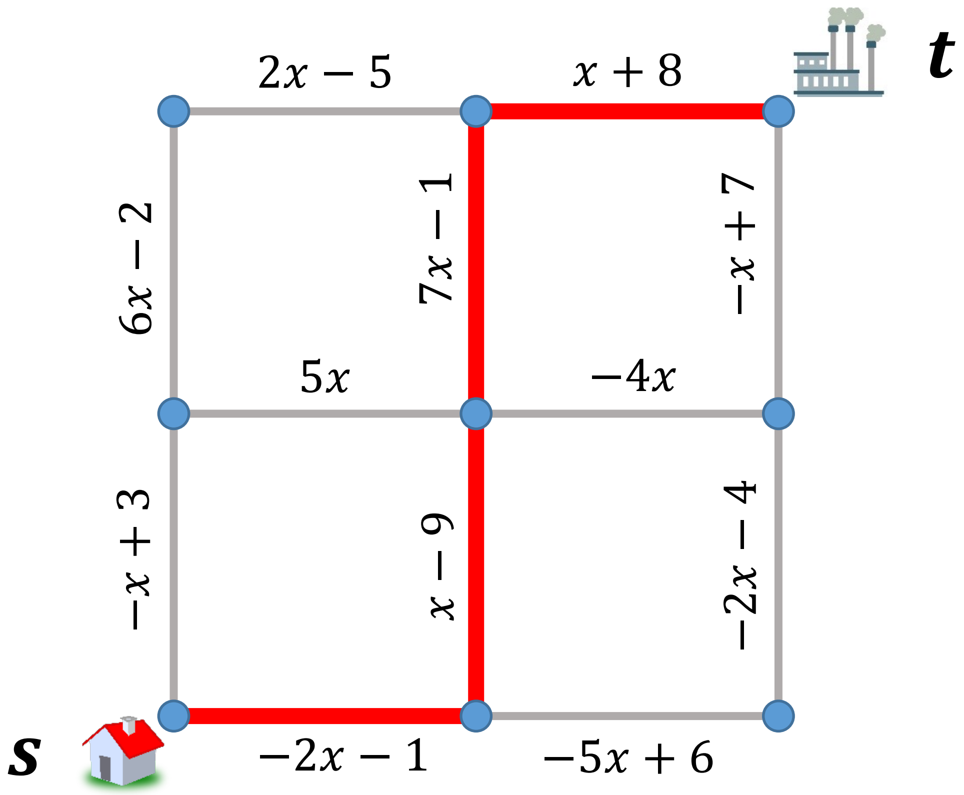
A typical non-lockdown workday



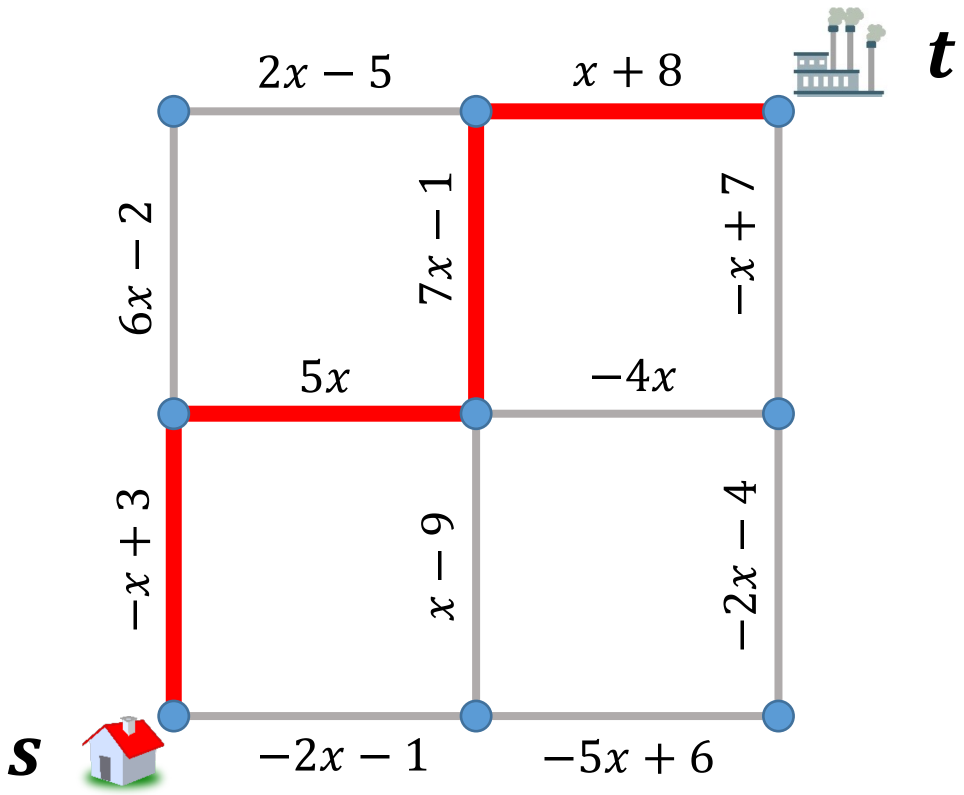
- Top view of city roadmap.
- Given: Arrival time at the end point of each road is a linear function of the time of departure from its start.
- Find: Shortest path from s to t at start time $x = 0$.

A typical non-lockdown workday



- Top view of city roadmap.
- Given: Travel time on each road is a linear function of time.
- Find: Shortest path from s to t at start time $x = 0$.

A typical non-lockdown workday

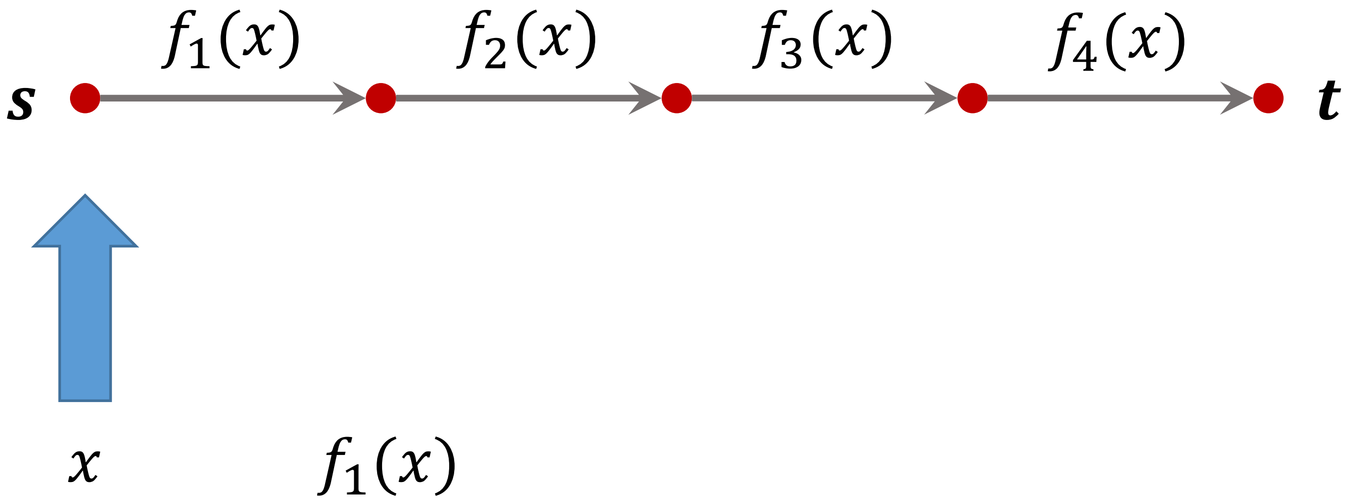


- Top view of city roadmap.
- Given: Travel time on each road is a linear function of time.
- Find: Shortest path from s to t at start time $x = 10$.

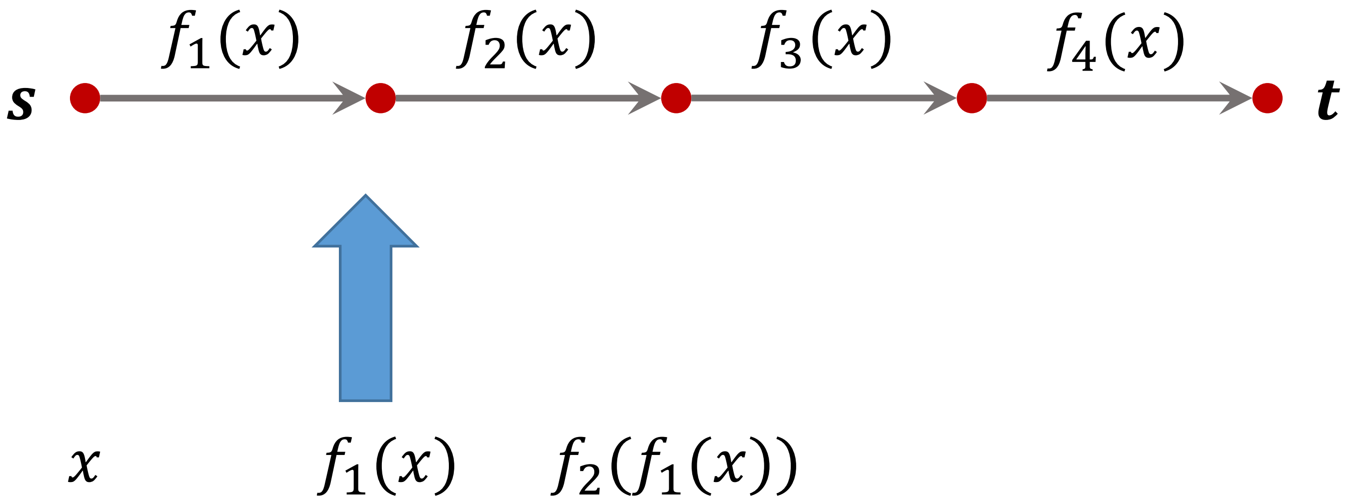
Edge weights: linear vs quadratic

- Earlier work: Computing with linear edge weights is “easy” or efficient.
- Our work: Computing with quadratic edge weights is “hard” or intractable.

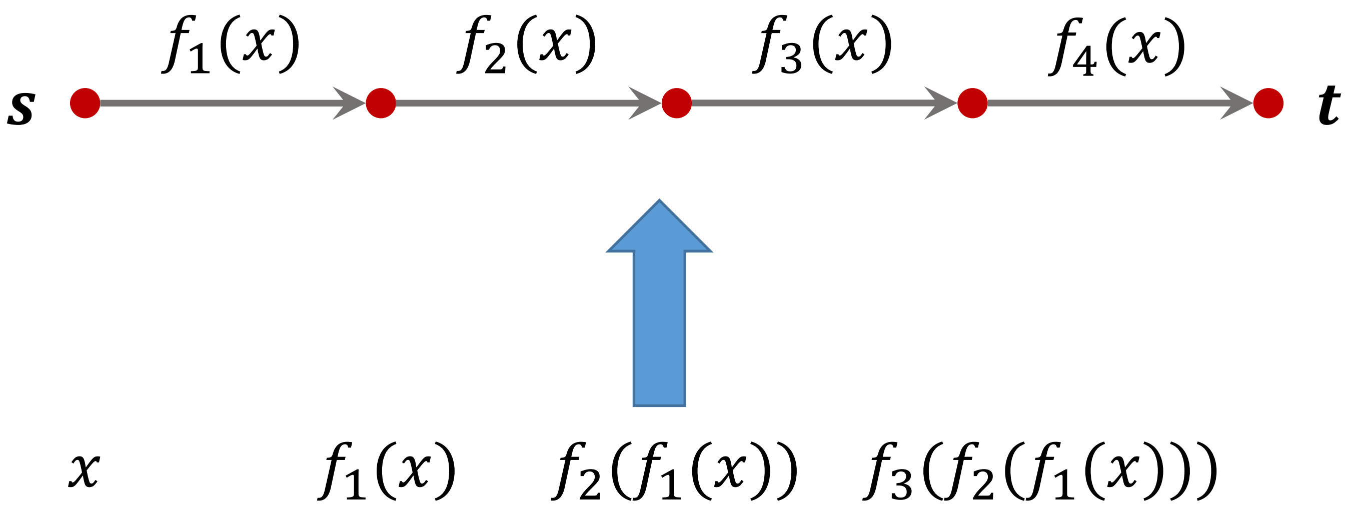
Edge weights: linear vs quadratic



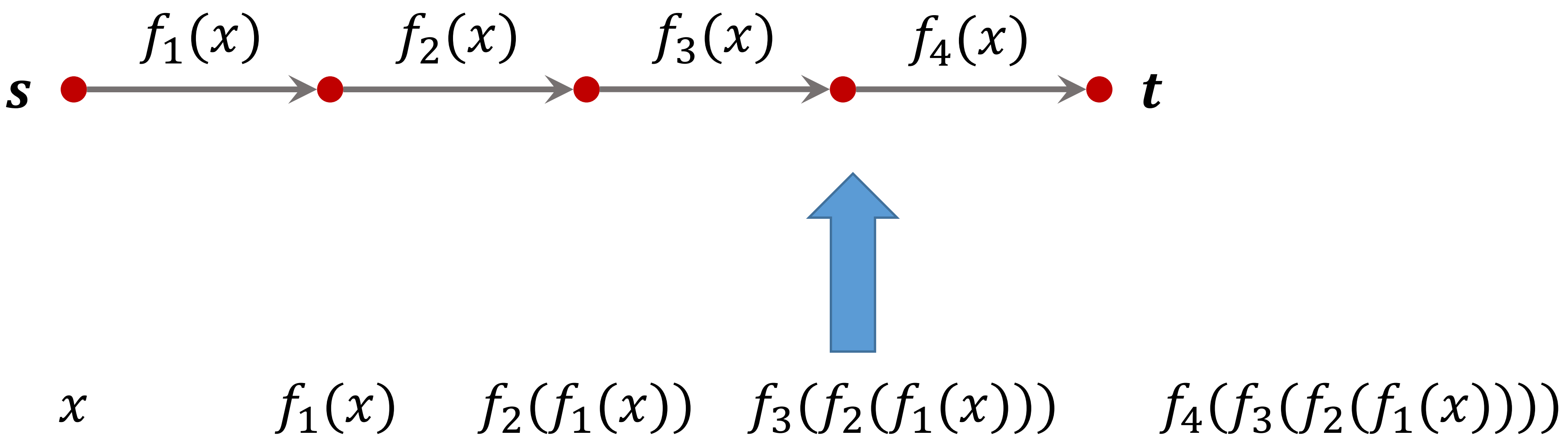
Edge weights: linear vs quadratic



Edge weights: linear vs quadratic



Edge weights: linear vs quadratic



If f_1, f_2, f_3, f_4 are linear, then $f_1 \circ f_2 \circ f_3 \circ f_4$ is also linear.

If f_1, f_2, f_3, f_4 are quadratic, then $f_1 \circ f_2 \circ f_3 \circ f_4$ can be a polynomial of degree 16.

Two settings: classical, pre-processing

- Finding the shortest path in real time: given a graph and a time x, find the shortest path to reach t, when departing from s at time x.

- In most practical scenarios, the layout of the road does not change on a day-to-day basis. Thus, we can pre-process the graph and store all the relevant information beforehand.

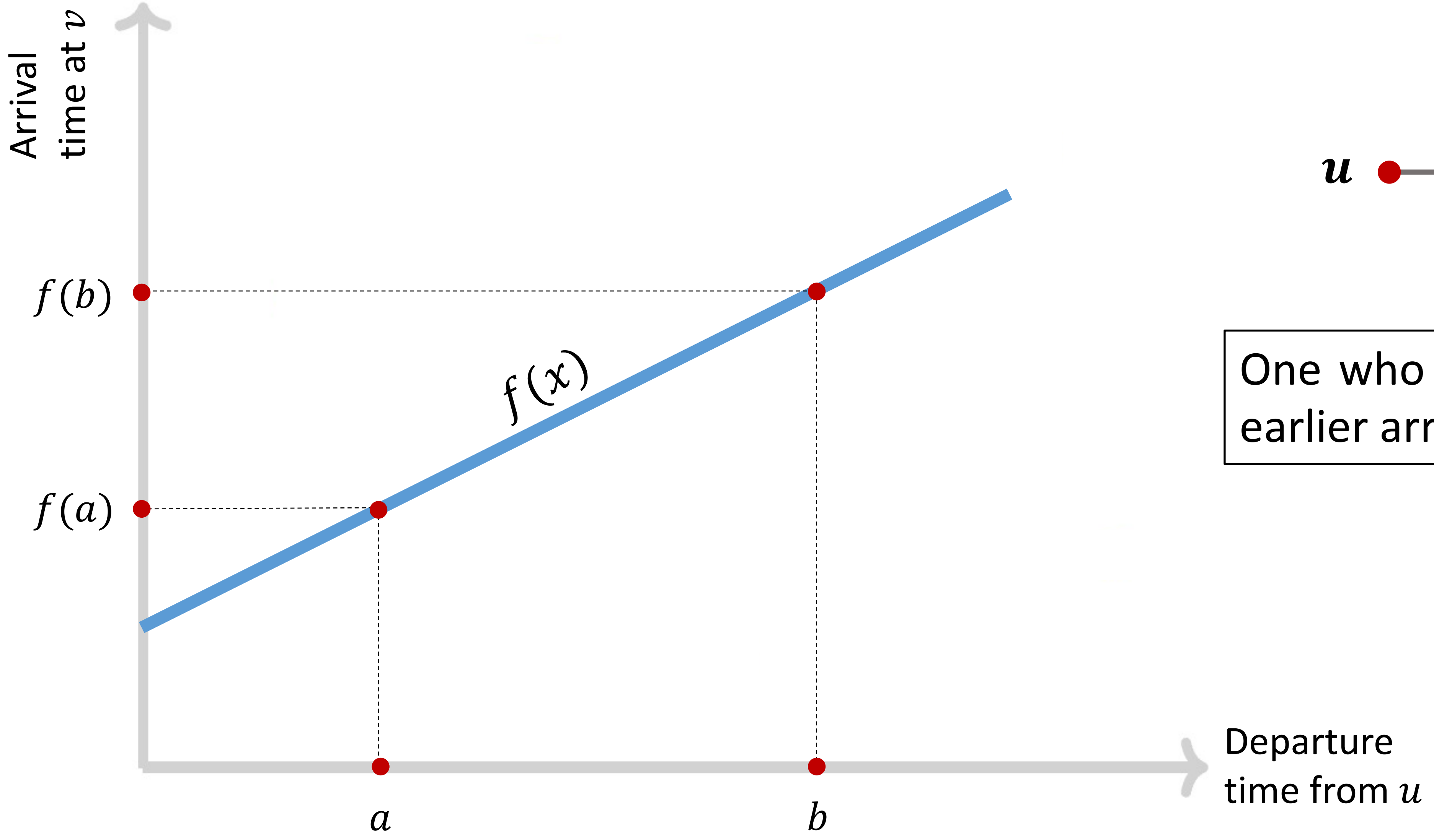
- Storage and retrieval of shortest paths: store all possible shortest paths, and quickly retrieve the shortest path at a given start time x.

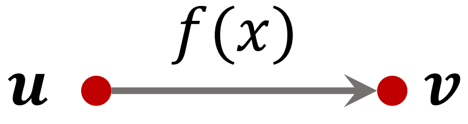
Earlier work

- Theorem [Foschini, Hershberger, Suri, 2011] (Classical setting) If the edge weights are monotonically increasing linear functions, then the shortest path can be computed in polynomial time.

- Theorem [Foschini, Hershberger, Suri, 2011] (Pre-processing setting) If the edge weights are monotonically increasing linear functions, then the shortest path can be retrieved in polylogarithmic time.

Why monotonically increasing?





One who departs from u earlier arrives at v earlier.

Braess' paradox

"This is not a real paradox but only a situation which is counter-intuitive." – Dietrich Braess, 1968

Scenario 1

Scenario 2

Braess' paradox

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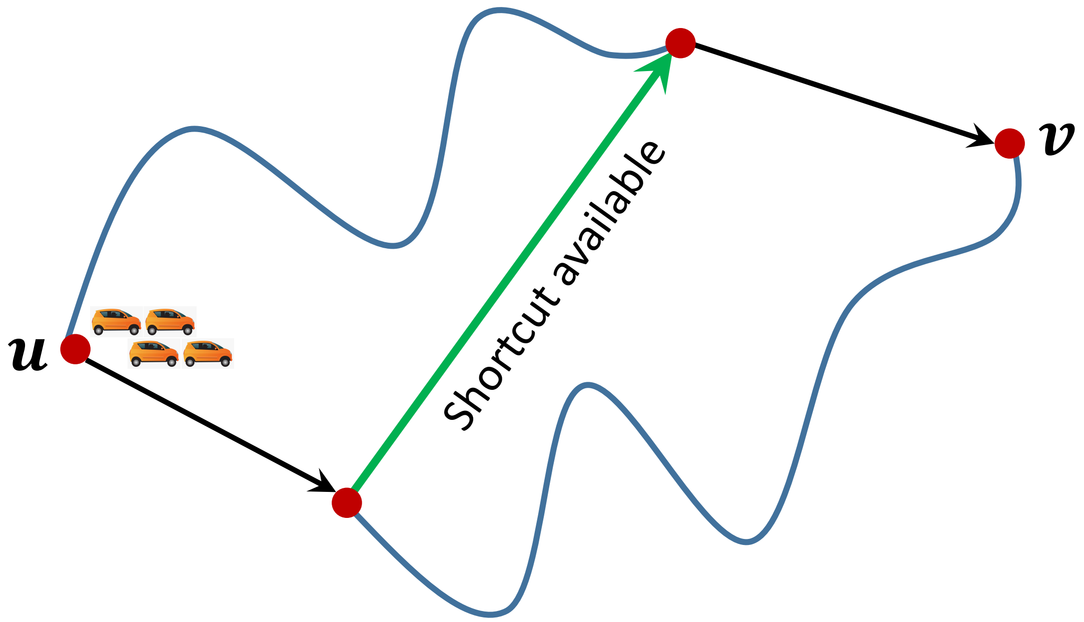

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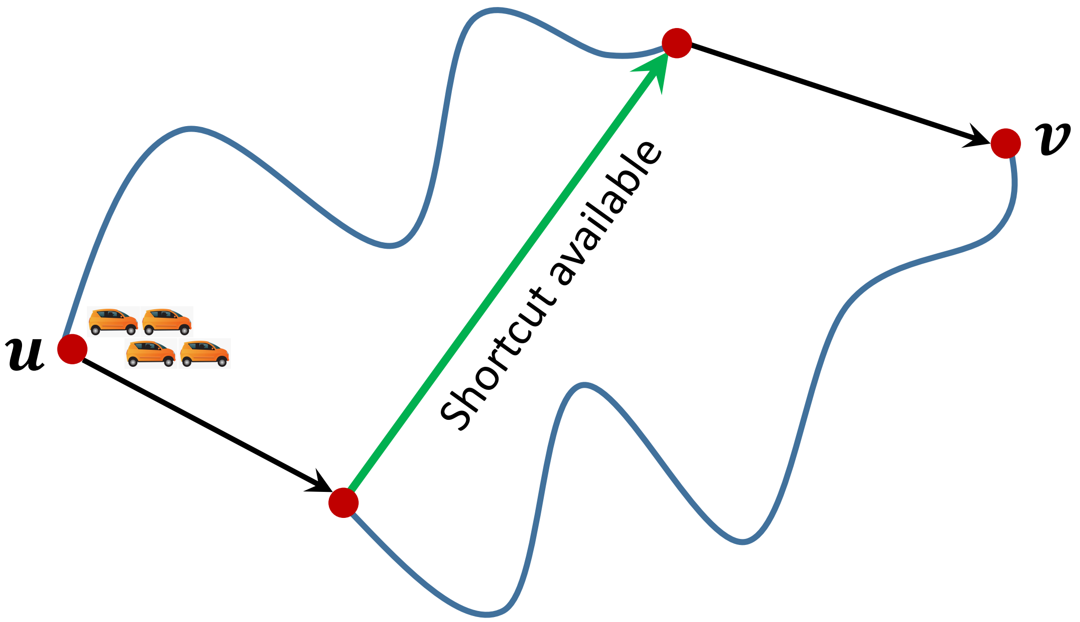
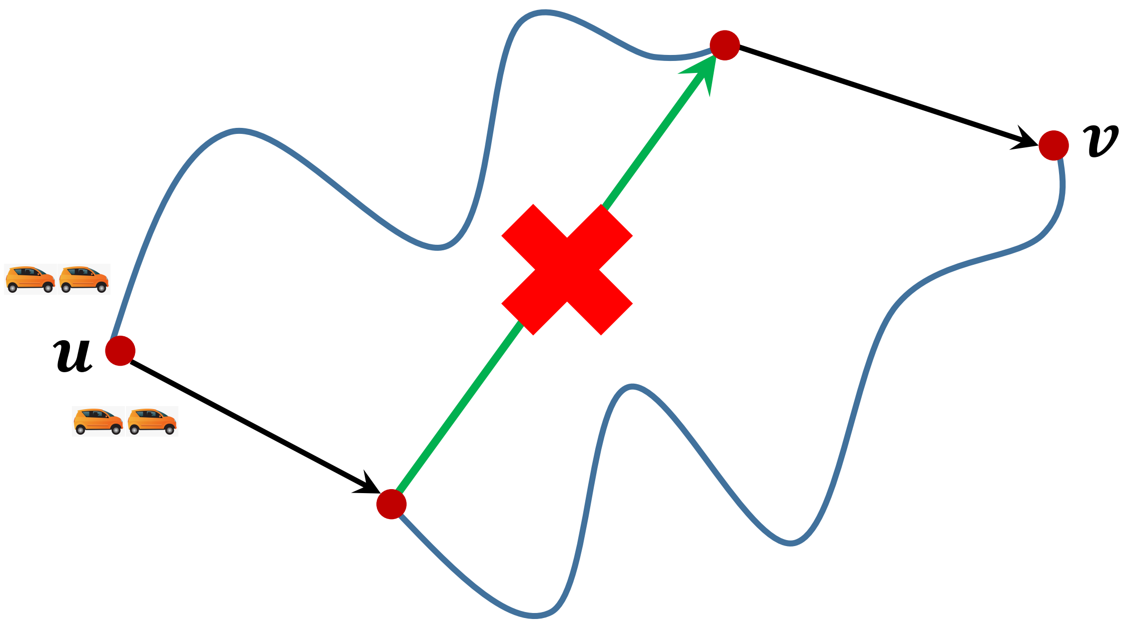


Scenario 1

More options available



Scenario 2

Less options available

Braess' paradox

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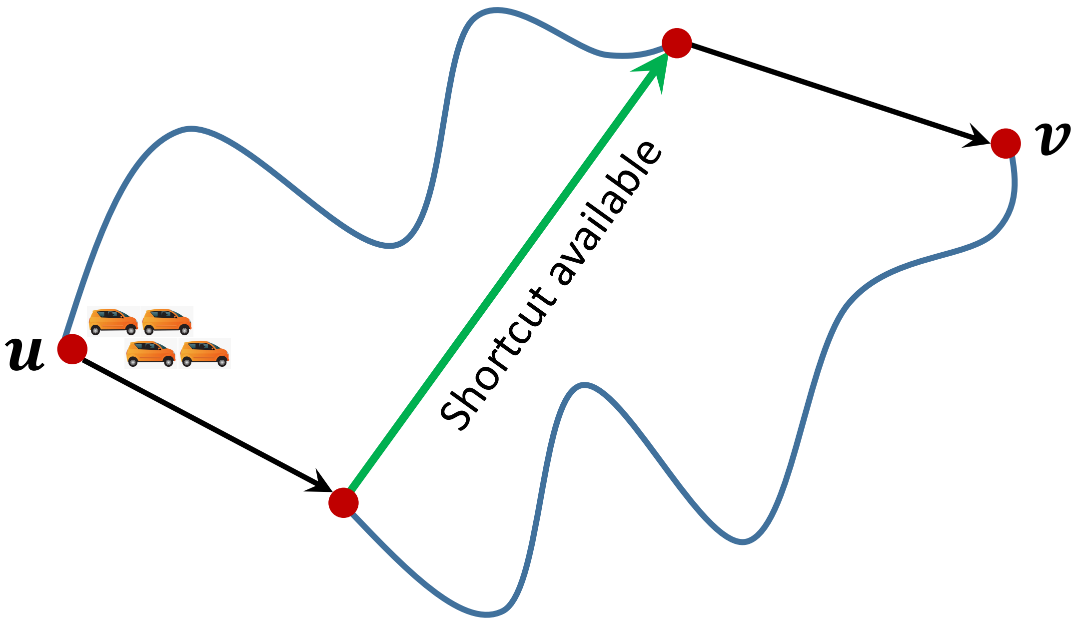
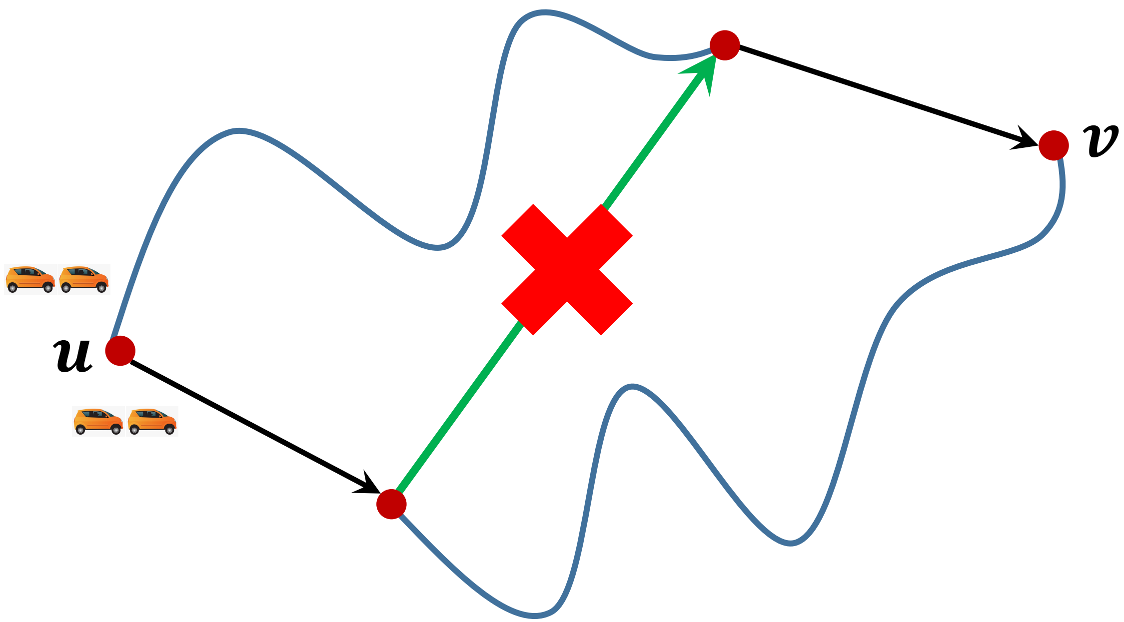


Scenario 1

More options available



Scenario 2

Less options available

Our results

Classical setting

- Theorem If the edge weights are linear functions, then the shortest path can be computed in polynomial time.

- Theorem If the edge weights are quadratic functions, then the shortest path cannot be computed in polynomial time, assuming **P≠NP**.

Our results

Pre-processing setting

- Theorem If the edge weights are linear functions, then the shortest path can always be retrieved in polylogarithmic time.

- Theorem If the edge weights are quadratic functions, then there are graphs in which the shortest path cannot be retrieved in sublinear time.

Thank You!