Monotone Classes Beyond VNP

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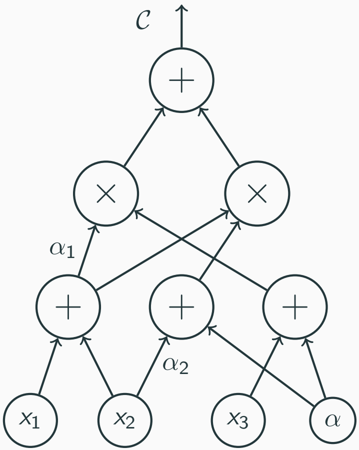
Tel Aviv University

December 18, 2023

Algebraic Classes



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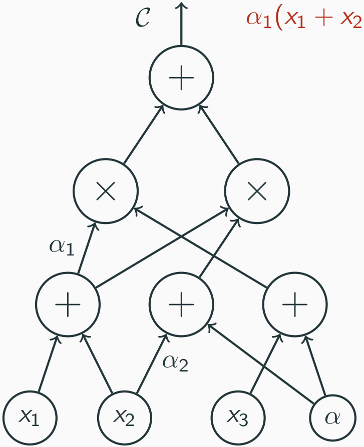

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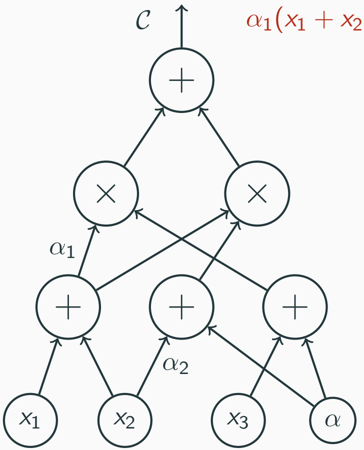

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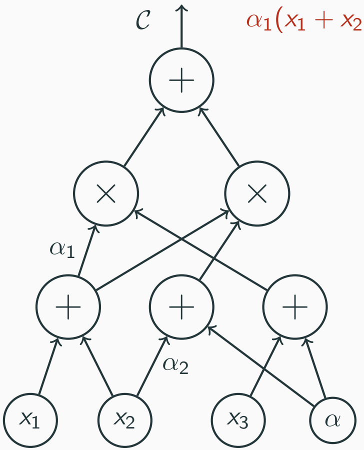

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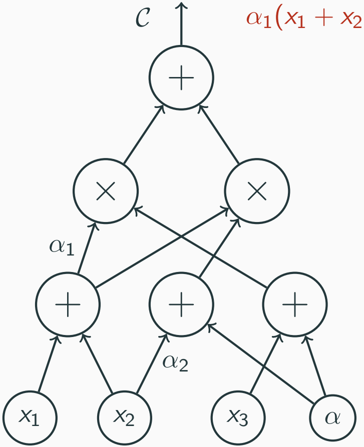

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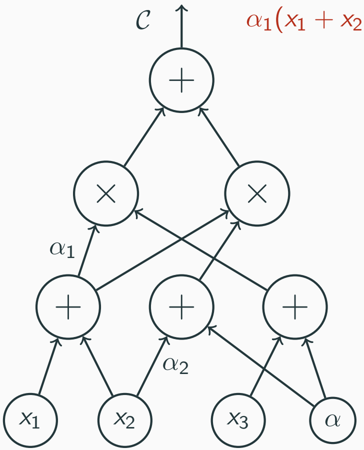

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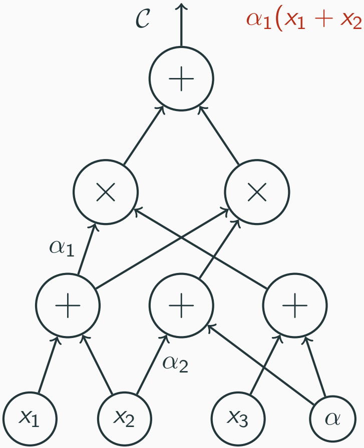

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What about classes beyond VNP?

The Class VPSPACE

[Koiran-Perifel]: $\{f_n\}_n$ is in $\mathsf{VPSPACE}^0$ if the following language is in PSPACE/ poly.

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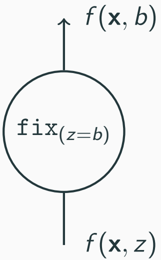

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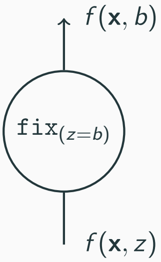

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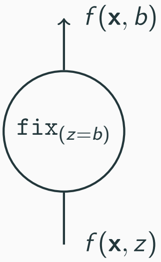

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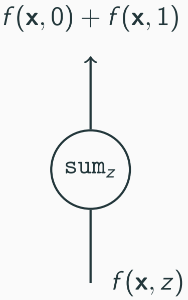

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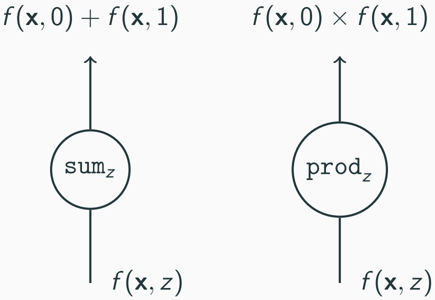

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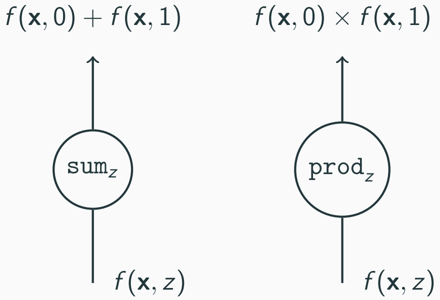

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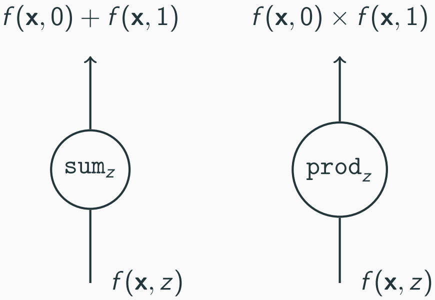

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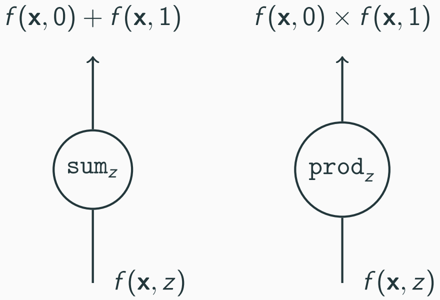

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Can we prove lower bounds in the monotone setting?

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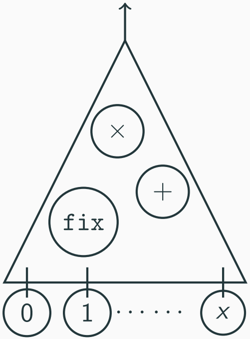

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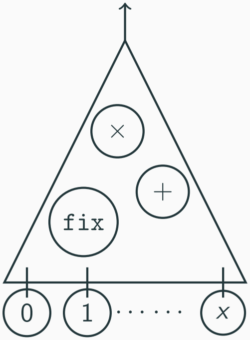

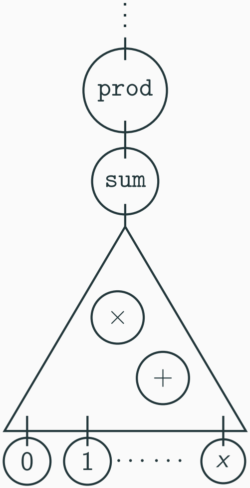

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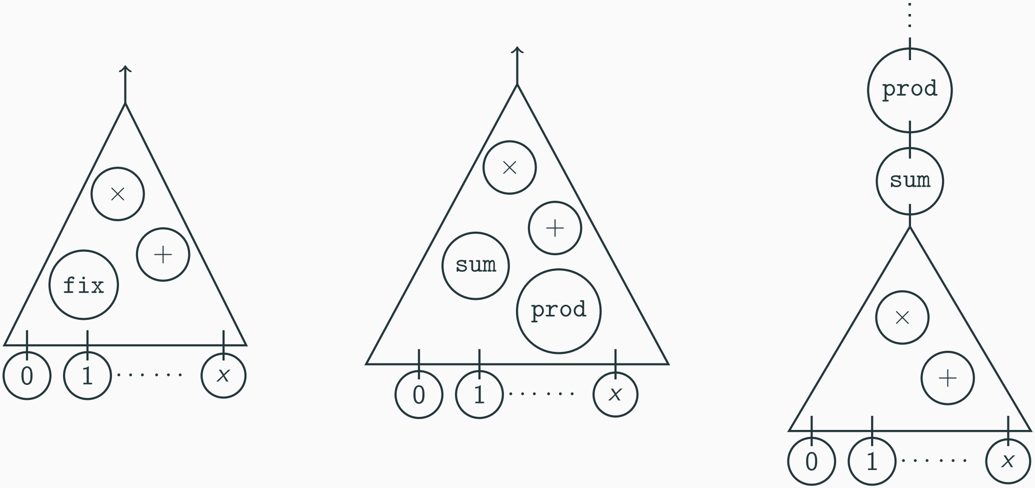
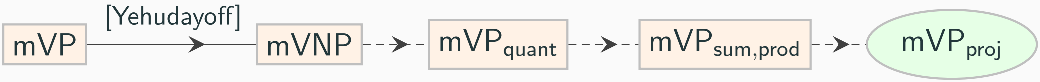


Our Results

Polynomials have bounded degree throughout



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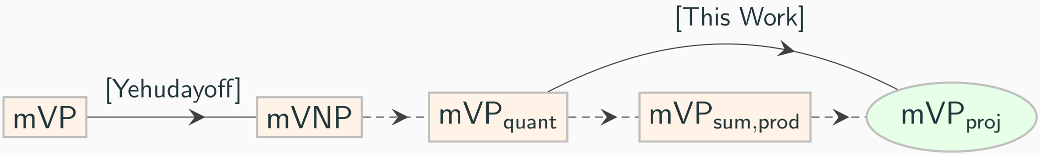

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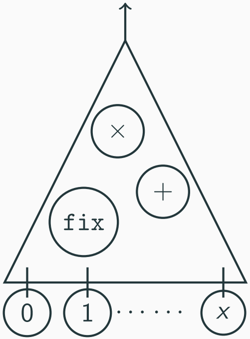

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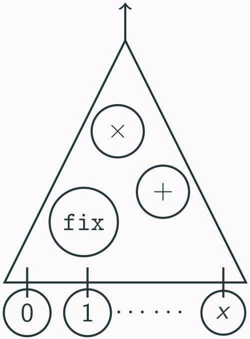

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Upper Bound for Permanent

$$P(\mathbf{x}, \mathbf{y}) := \sum_{\sigma \in S_n} x_{1,\sigma(1)} \cdots x_{n,\sigma(n)}.$$

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Open Questions I

- If $f \in \mathrm{mVP}_{\mathrm{quant}}$, then

$$f(\mathbf{x}) = \sum_{\mathbf{b} \in \{0,1\}^{|\mathbf{w}|}} A_f(\mathbf{w} = \mathbf{b}) \cdot g_f(\mathbf{x}, \mathbf{w} = \mathbf{b})$$

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- $\mathrm{VPH} = \mathtt{Q}_1 \mathtt{Q}_2 \cdots \mathtt{Q}_m \quad \mathcal{C}(\mathbf{x}, \mathbf{z})$ for constantly many alternations?

  Can we show that $\mathrm{VP} = \mathrm{VNP}$ implies that $\mathrm{VPH} = \mathrm{VP}$?

Open Questions II

τ**-conjecture for Newton polytopes** [Koiran-Portier-Tavenas-Thomassé]
Suppose $f(x, y)$ is a bivariate polynomial that can be written as $\sum_{i\in[s]} \prod_{j\in[r]} T_{i,j}(x, y)$, where each $T_{i,j}$ has sparsity at most p. Then the Newton polygon of f has poly(s, r, p) vertices.

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They give examples of transparent polynomials with exponential support.

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Polynomials that can be projected to bivariates in such a way that all of their monomials become vertices of the resulting Newton polygon.

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Can we extend this to $\mathrm{mVP}_{\mathrm{proj}}$?

Thank you!