The CHSH Game

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Alice and Bob play a game

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Game Play

- Alice and Bob go into separate rooms

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- Alice and Bob go into separate rooms
- They each flip a coin and note down the result.

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- Alice writes down a guess as to the result of Bob’s coin flip;

Alice and Bob play a game

Game Play

- Alice and Bob go into separate rooms
- They each flip a coin and note down the result.
- Alice writes down a guess as to the result of Bob’s coin flip; and Bob writes down a guess as to Alice’s flip.

Alice and Bob play a game

Game Play

- Alice and Bob go into separate rooms
- They each flip a coin and note down the result.
- Alice writes down a guess as to the result of Bob's coin flip; and Bob writes down a guess as to Alice's flip.

If both guesses are wrong then they both lose. Other wise they both win.

Alice and Bob play a game

Game Play

- Alice and Bob go into separate rooms
- They each flip a coin and note down the result.
- Alice writes down a guess as to the result of Bob's coin flip; and Bob writes down a guess as to Alice's flip.

Alice and Bob are teammates - they will win or lose the cash together.

If both guesses are wrong then they both lose. Other wise they both win.

Alice and Bob play a game

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- Alice writes down a guess as to the result of Bob's coin flip; and Bob writes down a guess as to Alice's flip.

Alice and Bob are teammates - they will win or lose the cash together. Before the game starts, they can talk to each other and agree on a strategy.

If both guesses are wrong then they both lose. Other wise they both win.

Alice and Bob play a game

Alice and Bob play a game

Alice flips a coin

Guesses result of Bob’s coin flip.

Alice and Bob play a game

Alice flips a coin

Guesses result of
Bob's coin flip.

Bob flips a coin

Guesses result of
Alice's coin flip.

Alice and Bob play a game

Alice flips a coin

Guesses result of
Bob's coin flip.

Bob flips a coin

Guesses result of
Alice's coin flip.

They win if at least one of them get it right.

Alice and Bob play a game

Alice flips a coin

Guesses result of
Bob's coin flip.

Bob flips a coin

Guesses result of
Alice's coin flip.

They win if at least one of them get it right.
What should they guess?

Alice and Bob play a game

Alice's Toss	Bob's Toss	Guesses that lead to Loss
Head	Head	Alice -> Tail & Bob -> Tail
Head	Tail	Alice -> Head & Bob -> Tail
Tail	Head	Alice -> Tail & Bob -> Head
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Case 1: Alice & Bob have same result

Case 2: Alice & Bob have different results

Alice and Bob play a game

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Case 1: Alice & Bob have same result

Case 2: Alice & Bob have different results

Winning Strategy

Alice guesses whatever her result is.

Bob guesses the opposite of what his result is.

Alice and Bob play a game

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Alice will assume this is the case.

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Case 1: Alice & Bob have same result
Alice will assume this is the case.

Case 2: Alice & Bob have different results
Bob will assume this is the case.

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Alice guesses whatever her result is.

Bob guesses the opposite of what his result is.

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Alice and Bob play another game

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Game Play

- Alice and Bob go into separate rooms

Alice and Bob play another game


Game Play

- Alice and Bob go into separate rooms
- Charlie goes into Alice's room and flips a coin. Let the result be X.

Alice and Bob play another game

Game Play

- Alice and Bob go into separate rooms
- Charlie goes into Alice's room and flips a coin. Let the result be X. He then asks Alice to choose either to keep the cash or not.

Alice and Bob play another game

Game Play

- Alice and Bob go into separate rooms
- Charlie goes into Alice’s room and flips a coin. Let the result be X. He then asks Alice to choose either to keep the cash or not.
- Charlie goes into Bob’s room and flips a coin. Let the result be Y. He then asks Bob to choose either to keep the cash or not.

Alice and Bob play another game

Game Play

- Alice and Bob go into separate rooms
- Charlie goes into Alice's room and flips a coin. Let the result be X. He then asks Alice to choose either to keep the cash or not.
- Charlie goes into Bob's room and flips a coin. Let the result be Y. He then asks Bob to choose either to keep the cash or not.

If X=Y=Head, they both win if and only if they make opposite choices.

Alice and Bob play another game

If X=Y=Head, they both win if and only if they make opposite choices.

Otherwise, they both win if and only if they make the same choice.

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Alice and Bob play another game

If X=Y=Head, they both win if and only if they make opposite choices.

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- Alice and Bob go into separate rooms
- Charlie goes into Alice's room and flips a coin. Let the result be X. He then asks Alice to choose either to keep the cash or not.
- Charlie goes into Bob's room and flips a coin. Let the result be Y. He then asks Bob to choose either to keep the cash or not.

Alice and Bob are teammates - they will win or lose the cash together. Before the game starts, they can talk to each other and agree on a strategy.

Alice and Bob play another game

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
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Game Play

- Alice and Bob go into separate rooms
- Charlie goes into Alice's room and flips a coin. Let the result be X. He then asks Alice to choose either to keep the cash or not.
- Charlie goes into Bob's room and flips a coin. Let the result be Y. He then asks Bob to choose either to keep the cash or not.

Alice and Bob play another game



Charlie flips a coin

Asks Alice if she wants to keep the cash.



Charlie flips a coin

Asks Bob if he wants to keep the cash.

What should Alice and Bob have said?

Alice and Bob play another game

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
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Keep

Alice and Bob play another game

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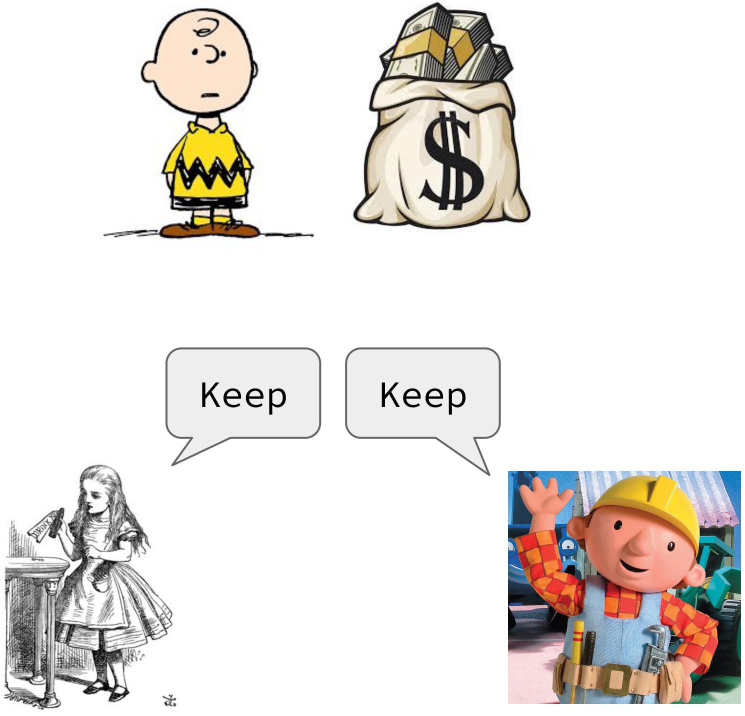

Alice and Bob win with probability 75%

Alice and Bob play another game

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Keep

Keep

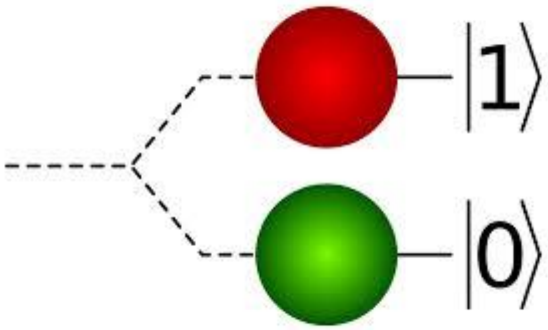
Bell's Inequality:
Alice and Bob can not come up with a better strategy even if they had access to shared random bits.

Alice and Bob win with probability 75%

WHAT IF THEY HAD ACCESS TO SHARED QUANTUM BITS?

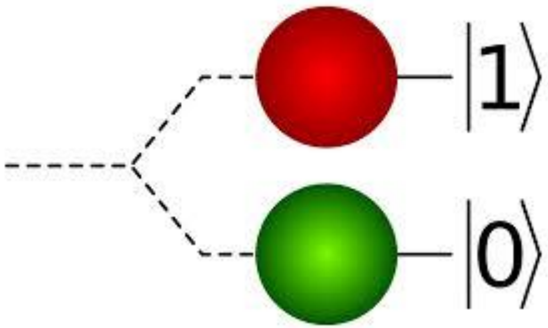
Quantum Bits (Qubits)

- A new data type with two basic values: |0> and |1>



Quantum Bits (Qubits)

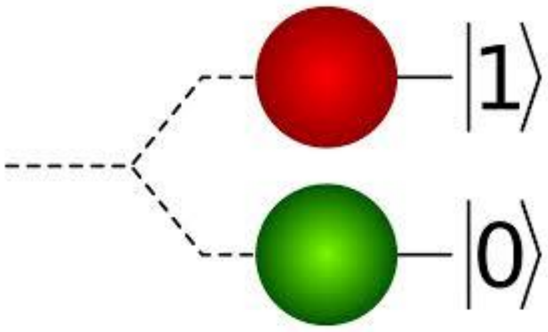
- A new data type with two basic values: |0> and |1>
- Magic of Quantum: A quantum bit can be both |0> and |1> until a measurement is made.



Quantum Bits (Qubits)

- A new data type with two basic values: |0> and |1>
- Magic of Quantum: A quantum bit can be both |0> and |1> until a measurement is made.
- A qubit has the form a|0> + b|1> for complex numbers a,b with |a|,|b| ∈ [0,1] such that

$$a^2 + b^2 = 1.$$



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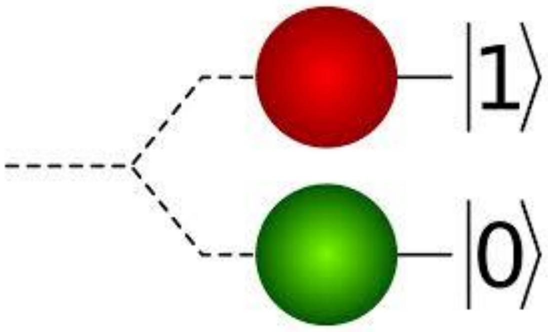
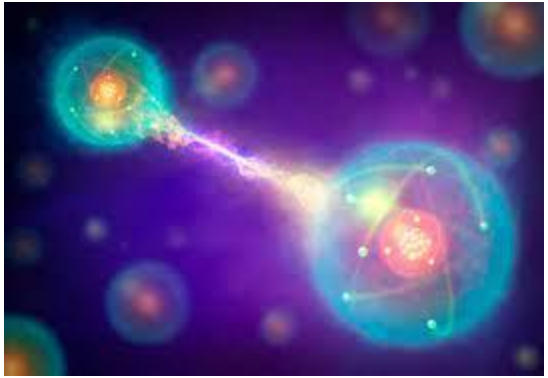


When such a qubit is measured, the probability of measuring |0> is a^2 and the probability of measuring |1> is b^2.

The EPR State

Two qubits are said to be entangled if they are not independent of each other.

Non-entangled: |00> = |0> X |0>



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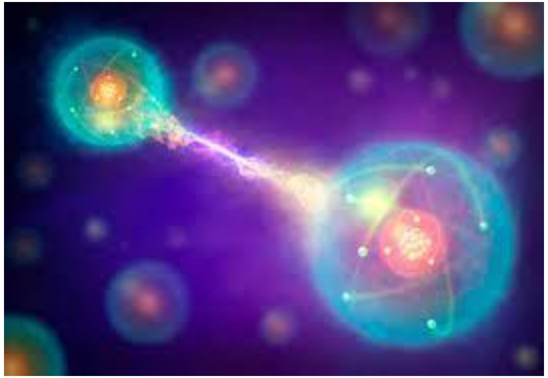
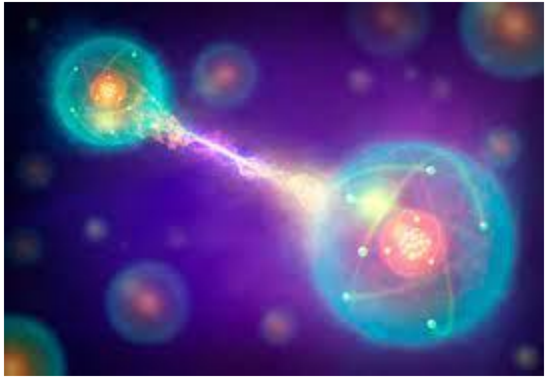


The EPR State

Two qubits are said to be entangled if they are not independent of each other.

Non-entangled: $|00\rangle = |0\rangle \times |0\rangle$

$$|00\rangle + |01\rangle = |0\rangle \times (|0\rangle + |1\rangle)$$



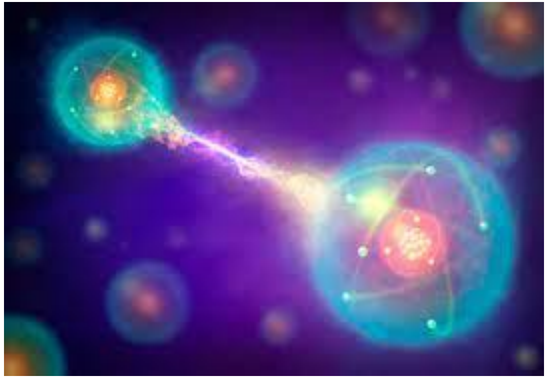
The EPR State

Two qubits are said to be entangled if they are not independent of each other.

Non-entangled: |00> = |0> × |0>

|00> + |01> = |0> × (|0> + |1>)

Entangled: |00> + |11>



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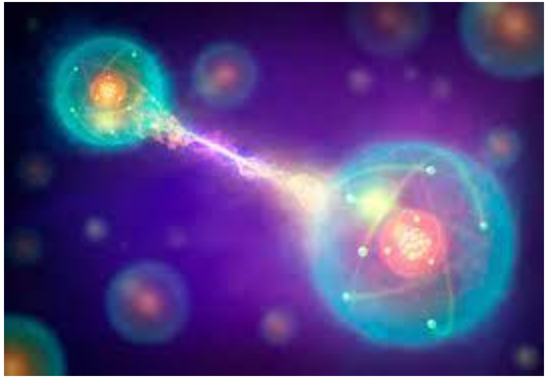
Can not be written as a product of two qubits.



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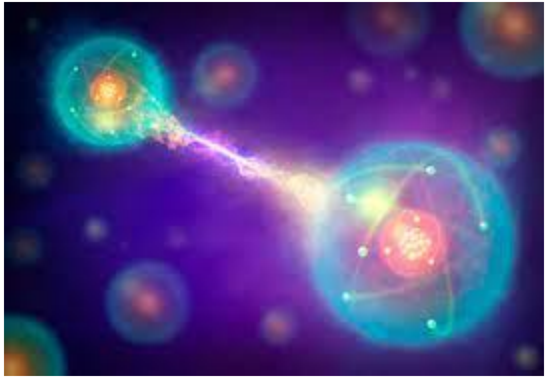

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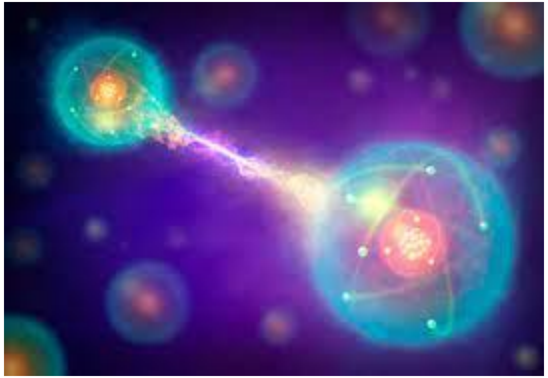

Note: We have not normalised the qubits.

Can Quantum Experiments be simulated using randomness?

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1. Alice and Bob create EPR pair: |00> + |11>

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1. Alice and Bob create EPR pair: |00> + |11>
2. Alice takes the first qubit with her and leaves Bob with the second qubit.

Can Quantum Experiments be simulated using randomness?

1. Alice and Bob create EPR pair: |00> + |11>
2. Alice takes the first qubit with her and leaves Bob with the second qubit.
3. Alice measures her qubit.

Can Quantum Experiments be simulated using randomness?

1. Alice and Bob create EPR pair: |00> + |11>
2. Alice takes the first qubit with her and leaves Bob with the second qubit.
3. Alice measures her qubit.
4. Bob measures his qubit.

Can Quantum Experiments be simulated using randomness?

1. Alice and Bob create EPR pair: |00> + |11>
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1. a = Random(0,1)
2. b = a

Can Quantum Experiments be simulated using randomness?

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2. Alice takes the first qubit with her and leaves Bob with the second qubit.
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1. a = Random(0,1)
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3. Alice takes A without looking at it and leaves B with Bob.

Can Quantum Experiments be simulated using randomness?

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Can Quantum Experiments be simulated using randomness?

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2. Alice takes the first qubit with her and leaves Bob with the second qubit.
3. Alice measures her qubit.
4. Bob measures his qubit.

1. a = Random(0,1)
2. b = a
3. Alice takes A without looking at it and leaves B with Bob.
4. Alice looks at a.
5. Bob looks at b.

Alice and Bob play another game




Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
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Alice and Bob win with probability 75% if the both say they will keep the cash.

Alice and Bob play another game




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Alice and Bob win with probability 75% if the both say they will keep the cash.

Bell’s Inequality

Alice and Bob can not come up with a better strategy even if they had access to random bits.

Alice and Bob play another game




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Alice and Bob win with probability 75% if the both say they will keep the cash.

Bell's Inequality

Alice and Bob can not come up with a better strategy even if they had access to random bits.

If Alice and Bob have access to an EPR pair, then there is a strategy that allows them to win with probability at least 80%

Playing around with the EPR Pair

Toggling from a Distance

```
Toggle(|0>) = |1>.
Toggle(|1>) = |0>.
```

Toggling from a Distance

Toggle(|0>) = |1>.
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Suppose Alice and Bob have an EPR pair: |00> + |11>

Toggling from a Distance

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Suppose Alice and Bob have an EPR pair: |00> + |11>

Alice takes the first qubit with her and leaves Bob with the second qubit.

Toggling from a Distance

Toggle(|0>) = |1>.
Toggle(|1>) = |0>.

Suppose Alice and Bob have an EPR pair: |00> + |11>

Alice takes the first qubit with her and leaves Bob with the second qubit.

State if Alice toggles her qubit: |10> + |01>

Toggling from a Distance

Toggle(|0>) = |1>.
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State if Alice toggles her qubit: |10> + |01>

State if Bob toggles his qubit: |01> + |10>

Toggling from a Distance

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Suppose Alice and Bob have an EPR pair: |00> + |11>

Alice takes the first qubit with her and leaves Bob with the second qubit.

State if Alice toggles her qubit: |10> + |01>

State if Bob toggles his qubit: |01> + |10>

It is as though Alice can toggle Bob's qubit by toggling her own qubit since they are equivalent operations.

Rotations

$$Rot_{\theta}(|0\rangle) = \cos(\theta)|0\rangle + \sin(\theta)|1\rangle$$

$$Rot_{\theta}(|1\rangle) = -\sin(\theta)|0\rangle + \cos(\theta)|1\rangle$$

Rotations

$$\text{Rot}_\theta(|0>) = \cos(\theta)|0> + \sin(\theta)|1>$$

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Suppose Alice and Bob have an EPR pair: |00> + |11>

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Rotations

Suppose Alice and Bob have an EPR pair: $|00> + |11>$

Alice takes the first qubit with her and leaves Bob with the second qubit.

$$\mathrm{Rot}_\theta(|0>) = \cos(\theta)|0> + \sin(\theta)|1>$$

$$\mathrm{Rot}_\theta(|1>) = -\sin(\theta)|0> + \cos(\theta)|1>$$

State if Alice performs Rot_θ on her qubit: $\mathrm{Rot}_\theta(|0>)|0> + \mathrm{Rot}_\theta(|1>)|1>$

$$\cos(\theta)|00> + \sin(\theta)|10> -\sin(\theta)|01> + \cos(\theta)|11>$$

Rotations

Suppose Alice and Bob have an EPR pair: $|00\rangle + |11\rangle$

Alice takes the first qubit with her and leaves Bob with the second qubit.

$$\mathrm{Rot}_\theta(|0\rangle) = \cos(\theta)|0\rangle + \sin(\theta)|1\rangle$$

$$\mathrm{Rot}_\theta(|1\rangle) = -\sin(\theta)|0\rangle + \cos(\theta)|1\rangle$$

State if Alice performs Rot_θ on her qubit: $\mathrm{Rot}_\theta(|0\rangle)|0\rangle + \mathrm{Rot}_\theta(|1\rangle)|1\rangle$

$$\cos(\theta)|00\rangle + \sin(\theta)|10\rangle$$
$$-\sin(\theta)|01\rangle + \cos(\theta)|11\rangle$$

State if Bob performs Rot_θ on his qubit: $|0\rangle\mathrm{Rot}_\theta(|0\rangle) + |1\rangle\mathrm{Rot}_\theta(|1\rangle)$

$$\cos(\theta)|00\rangle + \sin(\theta)|01\rangle$$
$$-\sin(\theta)|10\rangle + \cos(\theta)|11\rangle$$

Rotations

Suppose Alice and Bob have an EPR pair: |00> + |11>

Alice takes the first qubit with her and leaves Bob with the second qubit.

Note: cos(-θ)=cos(θ) and sin(-θ)=-sin(θ)

Rot_θ(|0>) = cos(θ)|0> + sin(θ)|1>

Rot_θ(|1>) = -sin(θ)|0> + cos(θ)|1>

State if Alice performs Rot_θ on her qubit: Rot_θ(|0>)|0> + Rot_θ(|1>)|1>

cos(θ)|00> + sin(θ)|10>
-sin(θ)|01> + cos(θ)|11>

State if Bob performs Rot_θ on his qubit: |0>Rot_θ(|0>) + |1>Rot_θ(|1>)

cos(θ)|00> + sin(θ)|01>
-sin(θ)|10> + cos(θ)|11>

Rotations

Suppose Alice and Bob have an EPR pair: |00> + |11>

Alice takes the first qubit with her and leaves Bob with the second qubit.

Note: cos(-θ)=cos(θ) and sin(-θ)=-sin(θ)

It is as though Alice can simulate Bob performing Rot_θ on his qubit by performing $Rot_{-\theta}$ on her own qubit.

$Rot_\theta(|0\rangle) = \cos(\theta)|0\rangle + \sin(\theta)|1\rangle$

$Rot_\theta(|1\rangle) = -\sin(\theta)|0\rangle + \cos(\theta)|1\rangle$

State if Alice performs Rot_θ on her qubit: $Rot_\theta(|0\rangle)|0\rangle + Rot_\theta(|1\rangle)|1\rangle$

$\cos(\theta)|00\rangle + \sin(\theta)|10\rangle$
$-\sin(\theta)|01\rangle + \cos(\theta)|11\rangle$

State if Bob performs Rot_θ on his qubit: $|0\rangle Rot_\theta(|0\rangle) + |1\rangle Rot_\theta(|1\rangle)$

$\cos(\theta)|00\rangle + \sin(\theta)|01\rangle$
$-\sin(\theta)|10\rangle + \cos(\theta)|11\rangle$

Law of Quantum Transformations

$2^k \times 2^k$ Unitary Matrices

exactly capture all physically possible transformations

that can be performed on k-length qubits.

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Law of Quantum Transformations

$2^k \times 2^k$ Unitary Matrices

exactly capture all physically possible transformations

that can be performed on k-length qubits.

|0> is equivalent to $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ |1> is equivalent to $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Toggle(|x>) ≍ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Law of Quantum Transformations

$2^k \times 2^k$ Unitary Matrices

exactly capture all physically possible transformations

that can be performed on k-length qubits.

$|0\rangle$ is equivalent to $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $|1\rangle$ is equivalent to $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\text{Toggle}(|x\rangle) \asymp \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\text{Rot}_\theta(|0\rangle) \asymp \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$

A better Quantum Strategy

Alice and Bob play another game




Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob win with probability 75% if the both say they will keep the cash.

Bell’s Inequality

Alice and Bob can not come up with a better strategy even if they had access to random bits.

Alice and Bob play another game

   

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob win with probability 75% if the both say they will keep the cash.

Bell's Inequality

Alice and Bob can not come up with a better strategy even if they had access to random bits.

If Alice and Bob have access to an EPR pair, then there is a strategy that allows them to win with probability at least 80%

Alice and Bob play The CHSH Game

Coin Toss Result (Alice)	**Coin Toss Result (Bob)**	**Winning Situations**
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep



Quantum Strategy

ALICE AND BOB PLAY THE CHSH GAME

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep




Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

Alice and Bob play The CHSH Game

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep




<u>Quantum Strategy</u>

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by $(-\pi/8)$.

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

Alice and Bob play The CHSH Game

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep




Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by ($-\pi/8$).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by ($\pi/8$).

Alice and Bob play The CHSH Game

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep




Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Alice and Bob play The CHSH Game




Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Claim: With this strategy, Alice and Bob will win with probability at least 80%

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by $(-\pi/8)$.

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by $(\pi/8)$.

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Alice and Bob play The CHSH Game

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by $(-\pi/8)$.

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by $(\pi/8)$.

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Alice and Bob play The CHSH Game

Case 1: Coin Toss Result for Alice and Bob is Tail

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 1: Coin Toss Result for Alice and Bob is Tail

In this case, both Alice and Bob directly measure their qubits without performing any transformations.

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 1: Coin Toss Result for Alice and Bob is Tail

In this case, both Alice and Bob directly measure their qubits without performing any transformations.

Since their qubits are entangled, this would mean their answers would always match.

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by ($-\pi/8$).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by ($\pi/8$).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 1: Coin Toss Result for Alice and Bob is Tail

In this case, both Alice and Bob directly measure their qubits without performing any transformations.

Since their qubits are entangled, this would mean their answers would always match.

So in this case, Alice and Bob always win.

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 2: Coin Toss Result for Alice is Tail and for Bob is Head

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (-π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 2: Coin Toss Result for Alice is Tail and for Bob is Head

In this case, Alice measures her qubit without performing any transformations.

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (-π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 2: Coin Toss Result for Alice is Tail and for Bob is Head

In this case, Alice measures her qubit without performing any transformations.

But Bob measures his qubit after rotation by (π/8).

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 2: Coin Toss Result for Alice is Tail and for Bob is Head

In this case, Alice measures her qubit without performing any transformations.

But Bob measures his qubit after rotation by $(\pi/8)$. So the (unnormalised) state before measurement is

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by $(-\pi/8)$.

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by $(\pi/8)$.

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 2: Coin Toss Result for Alice is Tail and for Bob is Head

In this case, Alice measures her qubit without performing any transformations.

But Bob measures his qubit after rotation by (π/8). So the (unnormalised) state before measurement is

$$\cos(\pi/8)|00\rangle + \sin(\pi/8)|01\rangle$$
$$-\sin(\pi/8)|10\rangle + \cos(\pi/8)|11\rangle$$

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (-π/8).

She then measures her qubit. If it is 1, she responds "Keep". Else she responds "Not Keep".

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds "Keep". Else he responds "Not Keep".

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 2: Coin Toss Result for Alice is Tail and for Bob is Head

The (unnormalised) state before measurement is

$$\cos(\pi/8)|00\rangle + \sin(\pi/8)|01\rangle$$
$$-\sin(\pi/8)|10\rangle + \cos(\pi/8)|11\rangle$$

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 2: Coin Toss Result for Alice is Tail and for Bob is Head

The (unnormalised) state before measurement is

$$\cos(\pi/8)|00> + \sin(\pi/8)|01>$$
$$-\sin(\pi/8)|10> + \cos(\pi/8)|11>$$

So the probability that both answer consistently is

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by $(-\pi/8)$.

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by $(\pi/8)$.

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 2: Coin Toss Result for Alice is Tail and for Bob is Head

The (unnormalised) state before measurement is

$$\cos(\pi/8)|00\rangle + \sin(\pi/8)|01\rangle$$
$$-\sin(\pi/8)|10\rangle + \cos(\pi/8)|11\rangle$$

So the probability that both answer consistently is

$$\cos^2(\pi/8) >= .85$$

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds "Keep". Else she responds "Not Keep".

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds "Keep". Else he responds "Not Keep".

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 3: Coin Toss Result for Alice is Head and for Alice is Tail

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 3: Coin Toss Result for Alice is Head and for Alice is Tail

Exact same calculation as in Case 2 shows that the probability that both answer consistently is

$$\cos^2(\pi/8) >= .85$$

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 4: Coin Toss Result for Alice and Bob is Head

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by $(-\pi/8)$.

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by $(\pi/8)$.

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep

Alice and Bob play The CHSH Game

Case 4: Coin Toss Result for Alice and Bob is Head

In this case, both Alice and Bob measure their qubits after rotating them by $(-\pi/8)$, $(\pi/8)$ respectively.

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by $(-\pi/8)$.

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by $(\pi/8)$.

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep

Alice and Bob play The CHSH Game

Case 4: Coin Toss Result for Alice and Bob is Head

In this case, both Alice and Bob measure their qubits after rotating them by (-π/8), (π/8) respectively.

One can check that after these transformations, the probability of observing each of the basic states |00>, |01>, |10> and |11> is equal.

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (-π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep

Alice and Bob play The CHSH Game

Case 4: Coin Toss Result for Alice and Bob is Head

In this case, both Alice and Bob measure their qubits after rotating them by (−π/8), (π/8) respectively.

One can check that after these transformations, the probability of observing each of the basic states |00>, |01>, |10> and |11> is equal.

Thus, the probability of Alice and Bob answering differently is 0.5

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	**Coin Toss Result (Bob)**	**Winning Situations**
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep

Alice and Bob play The CHSH Game

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

<u>Quantum Strategy</u>

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Alice and Bob play The CHSH Game

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Win Probabilty is at least

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Alice and Bob play The CHSH Game

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Win Probabilty is at least 0.25×1




Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by $(-\pi/8)$.

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by $(\pi/8)$.

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Alice and Bob play The CHSH Game

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Win Probabilty is at least
$0.25 \times 1 + 0.5 \times 0.85$




Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by $(-\pi/8)$.

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by $(\pi/8)$.

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Alice and Bob play The CHSH Game




Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Win Probabilty is at least

$$0.25 \times 1 + 0.5 \times 0.85 + 0.25 \times 0.5$$

<u>Quantum Strategy</u>

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by $(-\pi/8)$.

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by $(\pi/8)$.

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Alice and Bob play The CHSH Game




Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Win Probabilty is at least
$0.25 \times 1 + 0.5 \times 0.85 + 0.25 \times 0.5 = 0.8$

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by ($-\pi/8$).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by ($\pi/8$).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

The CHSH Game Proves that there are experiments in which quantum Algorithms cannot be simulated by classical randomised algorithms.

THANK YOU !

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https://preronac.bitbucket.io/