ENTROPY

IN

ALGEBRAIC INDEPENDENCE TESTING

JR-FEST, ICTS-TIFR

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Algebraic Independence

Vector Space: $\mathbb{R}^3$ over $\mathbb{R}$.

$$v_1 = (1,0,1) \qquad v_2 = (0,1,0) \qquad v_3 = (1,2,1).$$

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Space of bi-variate polynomials over $\mathbb{C}$.

$$f_1 = x^2 \qquad f_2 = y^2 \qquad f_3 = xy$$

are algebraically dependent since $f_1 f_2 - f_3^2 = 0$.

Algebraic Independence

$f_1, f_2, \ldots, f_k \in \mathbb{F}[x_1, \ldots, x_n]$ are said to be algebraically dependent if there exists an $A \in \mathbb{F}[y_1, \ldots, y_k]$ such that $A(y_1, \ldots, y_k) \neq 0$ but $A(f_1, \ldots, f_k) = 0$.

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- algebraically dependent over $\mathbb{F}_p$.

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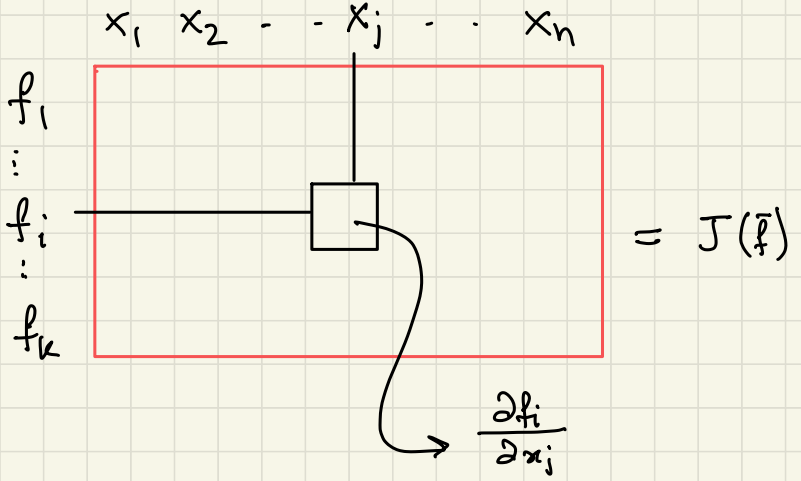

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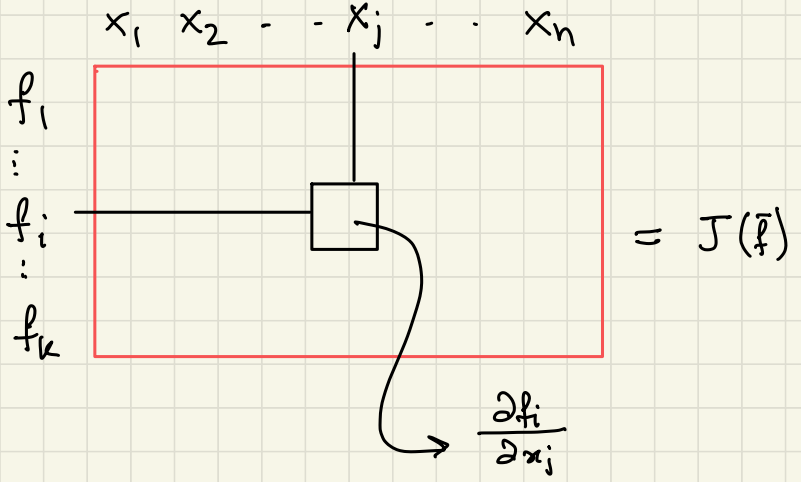
$f_1, \ldots, f_k \in \mathbb{F}[x_1, \ldots, x_n]$ are algebraically independent iff $J(\bar{f})$ has full rank.



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- Mittmann, Saxena, Scheiblechner (2014) showed that $AD(\mathbb{F}_q) \in NP^{\#P}$.
- Guo, Saxena, Sinhababu (2019) showed that $AD(\mathbb{F}_q) \in AM \cap co\text{-}AM$.

Testing Algebraic Independence in AM ∩ co-AM.

Core Observation: Let $f_1, \ldots, f_k \in \mathbb{F}_q[x_1, \ldots, x_n]$.

For any $(b_1, \ldots, b_k) \in \mathbb{F}_q$, let N_b be the number of solutions to the system of equations $\{f_i = b_i\}$.

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(1) For a random point $\bar{a} \in \mathbb{F}_q^n$

- if $f_1, \ldots, f_k$ are independent, then $N_{f(a)}$ is small.
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(2) For a random point $\bar{b} \in \mathbb{F}_q^k$
- if $f_1, \ldots, f_k$ are independent, then $N_b = 0$ for most b.
- if $f_1, \ldots, f_k$ are dependent, then $N_b \geq 1$ for many b.

Restating the Observation in terms of Entropy.

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- $\mathbb{F}$ is a field with $|\mathbb{F}| > 2c\, n d^{2n+1}$
- $f_1, \ldots, f_n \in \mathbb{F}^n \to \mathbb{F}^n$ are algebraically dependent
- $\deg(f_i) \le d \quad \forall i$
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Proof of (1)

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Choose $S = \mathrm{Supp}(Y) \setminus \mathrm{Supp}(X)$. $\Rightarrow \Pr_X(S) = 0$.

To Show: $|X - Y|_{TV} \geq c \cdot n \cdot \frac{d^{2n+1}}{|\mathbb{F}|}$.

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$$\leq d^n \cdot |\mathbb{F}|^{n-1} \left(\frac{d}{|\mathbb{F}|}\right)^n \Rightarrow |X-Y| \geq c \cdot n \cdot \frac{d^{2n+1}}{|\mathbb{F}|}$$

using the assumption on $|\mathbb{F}|$.

Proof of (2)

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For $i \in [n]$, define

$A_i \in \mathbb{F}[z, y_1, \ldots, y_n] \equiv$

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Claim:
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Estimating $|S|$:

Let $A_i = \sum_{j=0}^{d_i} A_{ij}(\bar{y}) \cdot z^j$

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$\Rightarrow |S| \geq (1-\delta) \cdot |\mathbb{F}|^n$ where $\delta = \frac{1}{2cd^n}$

Computing min-entropy of Y.

Let $\bar{b} \in f(S)$.

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$Y = f(U_S)$.

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Let $\bar{b} \in f(S)$.

$$N_{\bar{b}} = \{\bar{a} \in \mathbb{F}^n : f_i(\bar{a}) = b_i \ \forall i\}$$
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Bezout $\Rightarrow |\tilde{N}_{\bar{b}}| \leq \prod_{i=1}^{n} \deg(g_i)$ where $g_i = f_i(x) - b_i$

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let $\bar{b} \in f(S)$.

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where $|S| \geq \left(1 - \frac{1}{2cd^n}\right) \cdot |\mathbb{F}|^n$

Computing min-entropy of Y.

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$$\leq n \frac{d^{2n+1}}{|\mathbb{F}|}.$$

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