Magic or Mathematics ?!?

Prerona Chatterjee

NISER Bhubaneswar

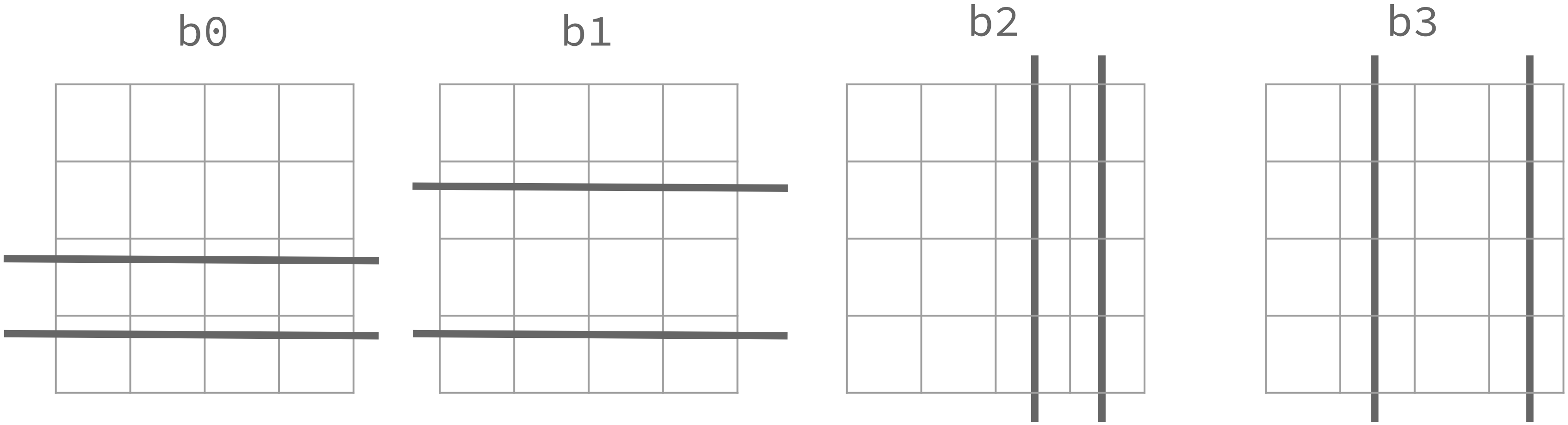
The Power of Two !!

Fill arbitrarily
with red/black.

The Power of Two!!

$B \equiv b0\ b1\ b2\ b3 \in \{0,1\}^4$

0 : #black is even
1 : #black is odd



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Want: Grid corresponding to
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\hline
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\end{array}$$

Ans: a0⊕c0 a1⊕c1 a2⊕c2 a3⊕c3

```
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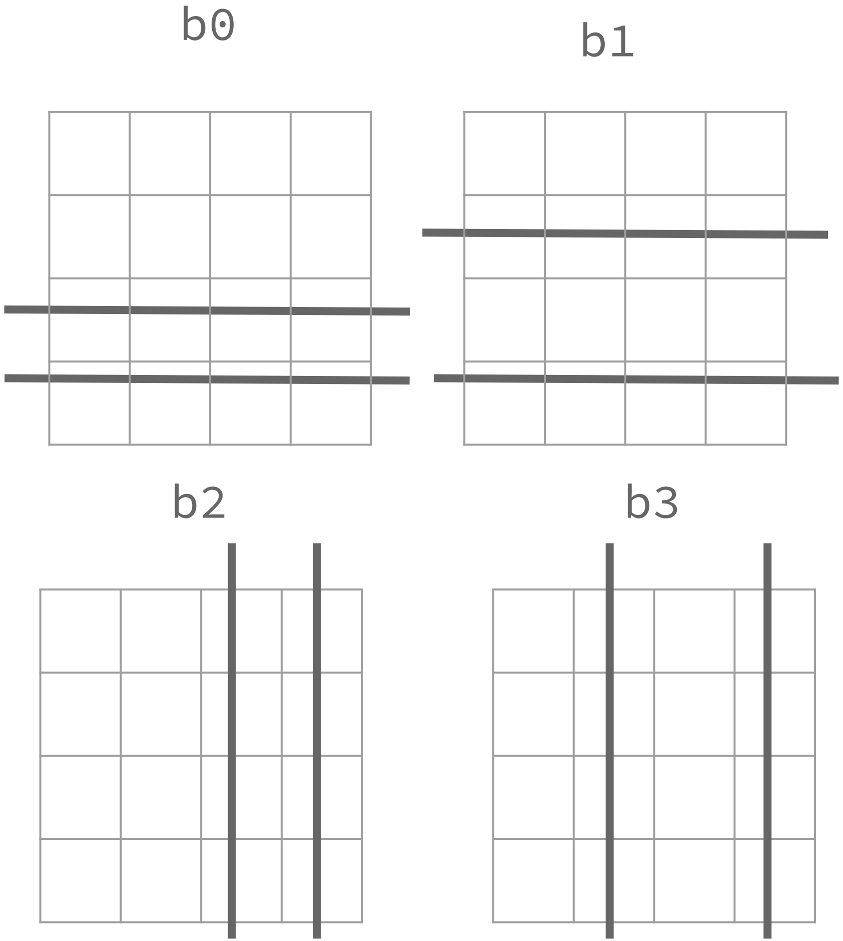

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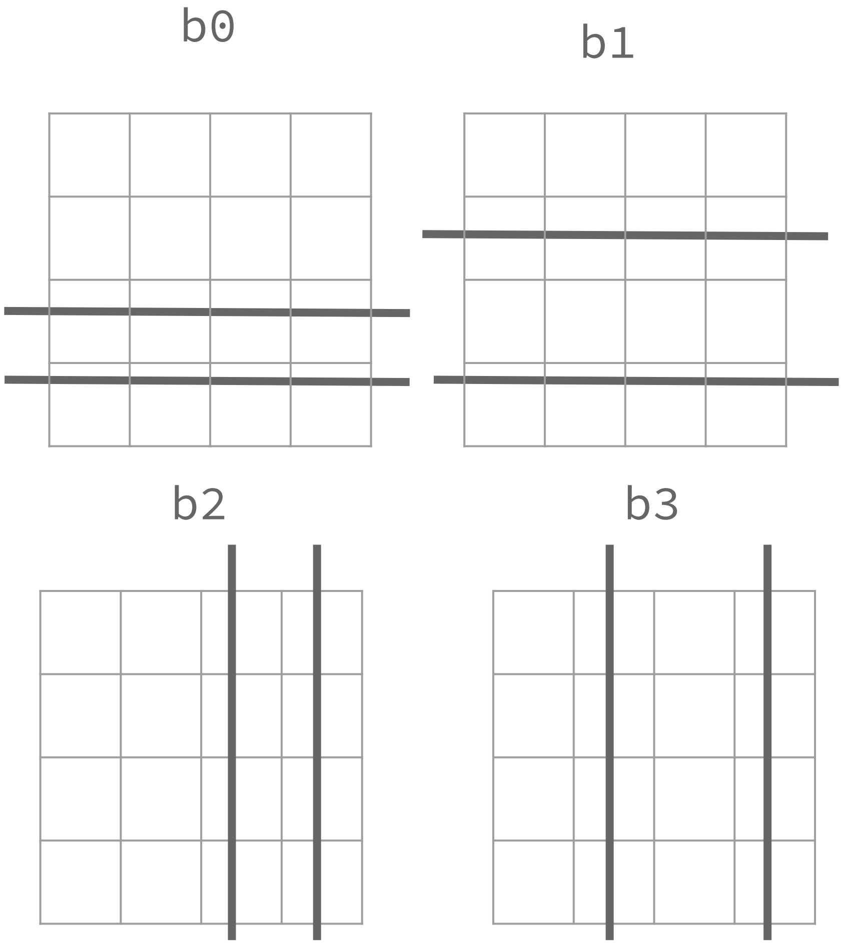

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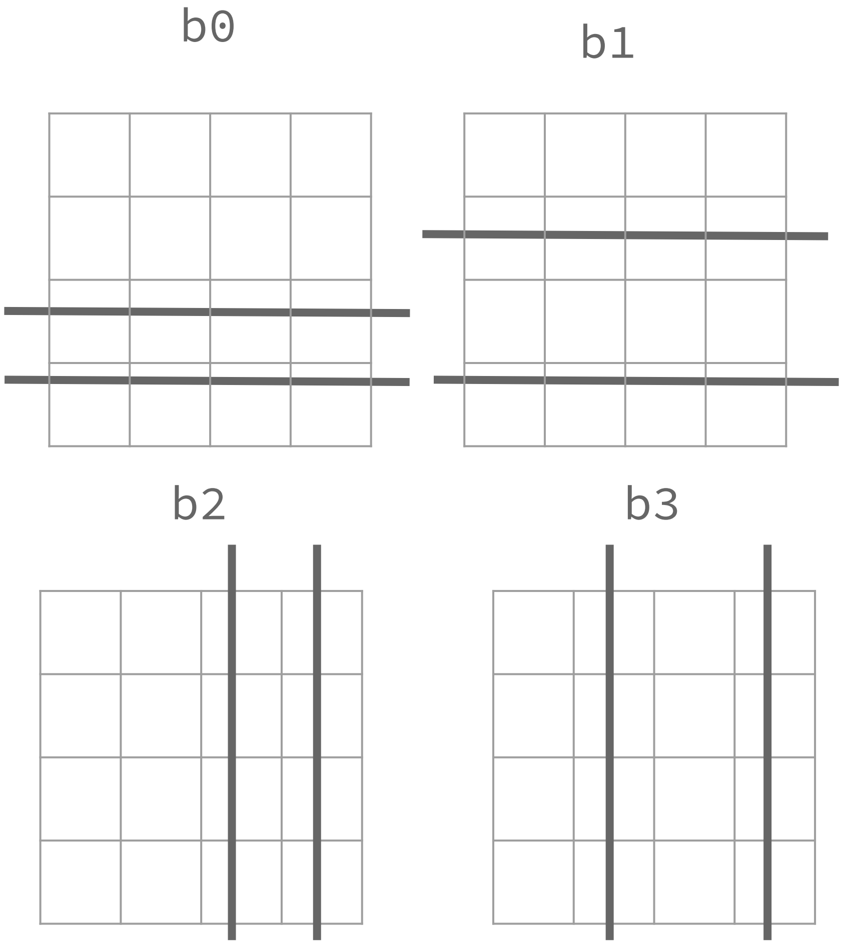
1100	1101	1110	1111



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Corruption of Data

- Weak Signal
- Cables affected
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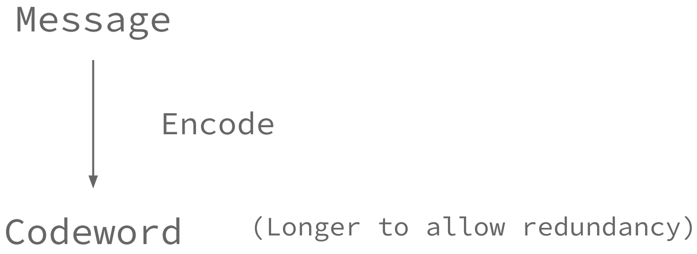

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Codeword (Longer to allow redundancy)

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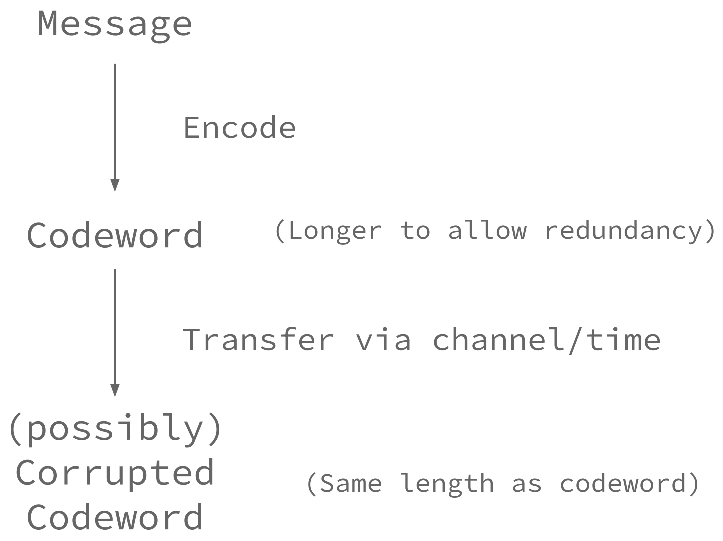

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(possibly) Corrupted Codeword

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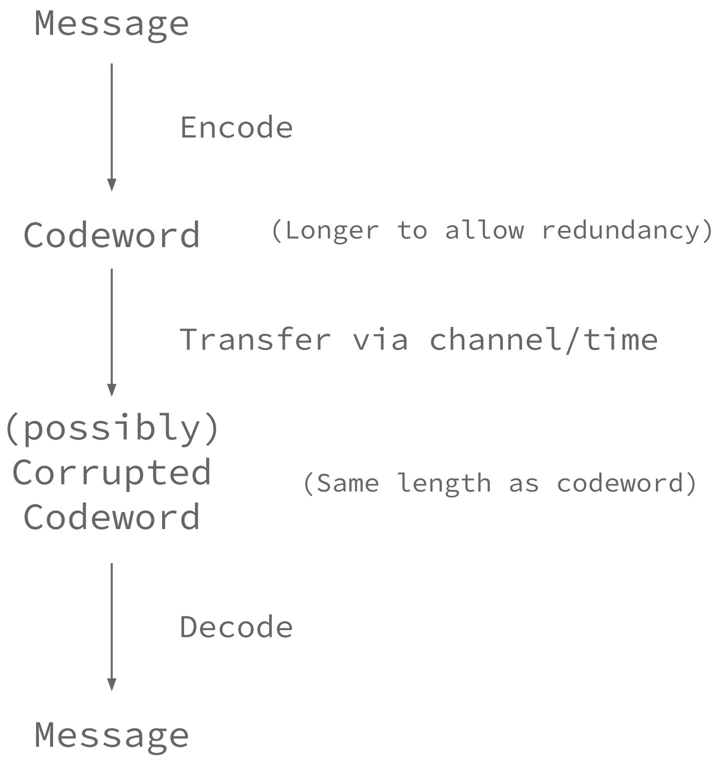

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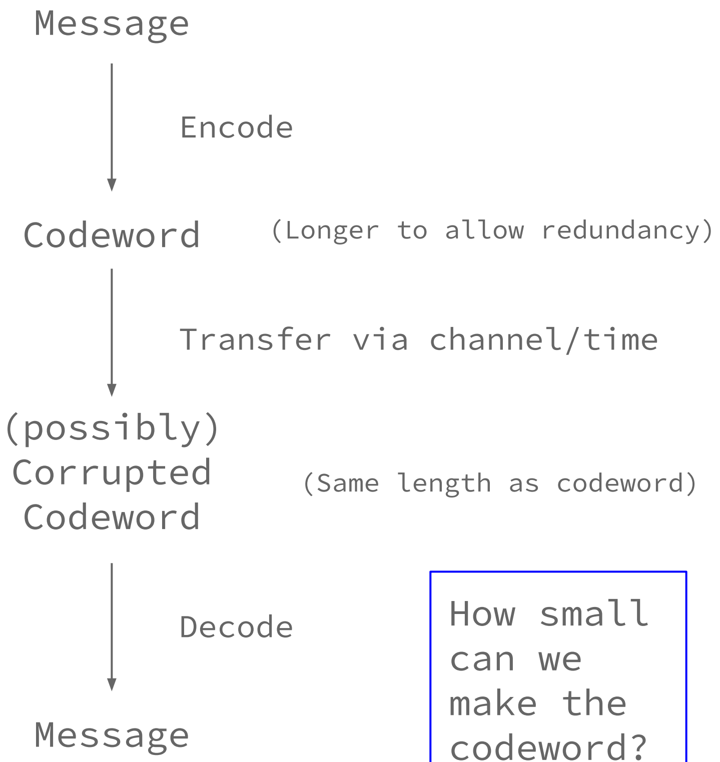

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How small can we make the codeword?

Sending Secret Messages!!

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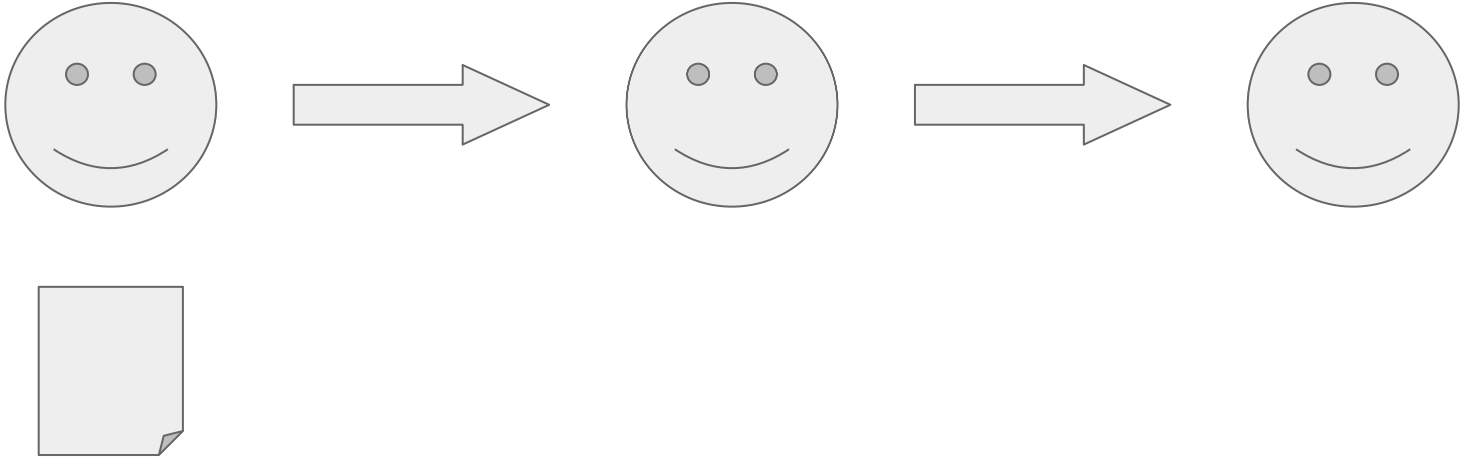

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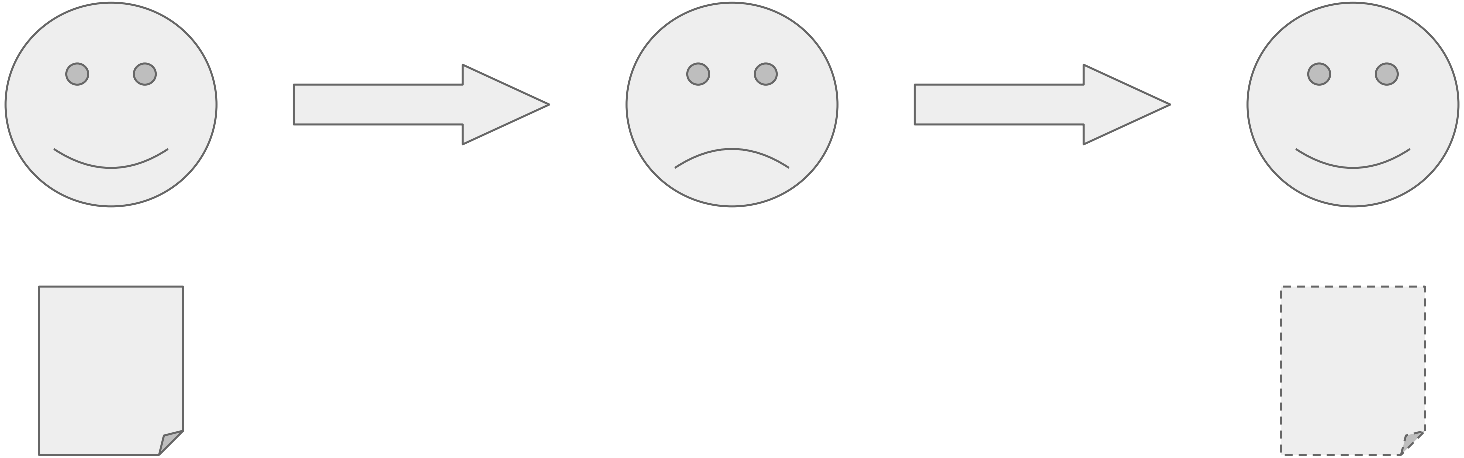

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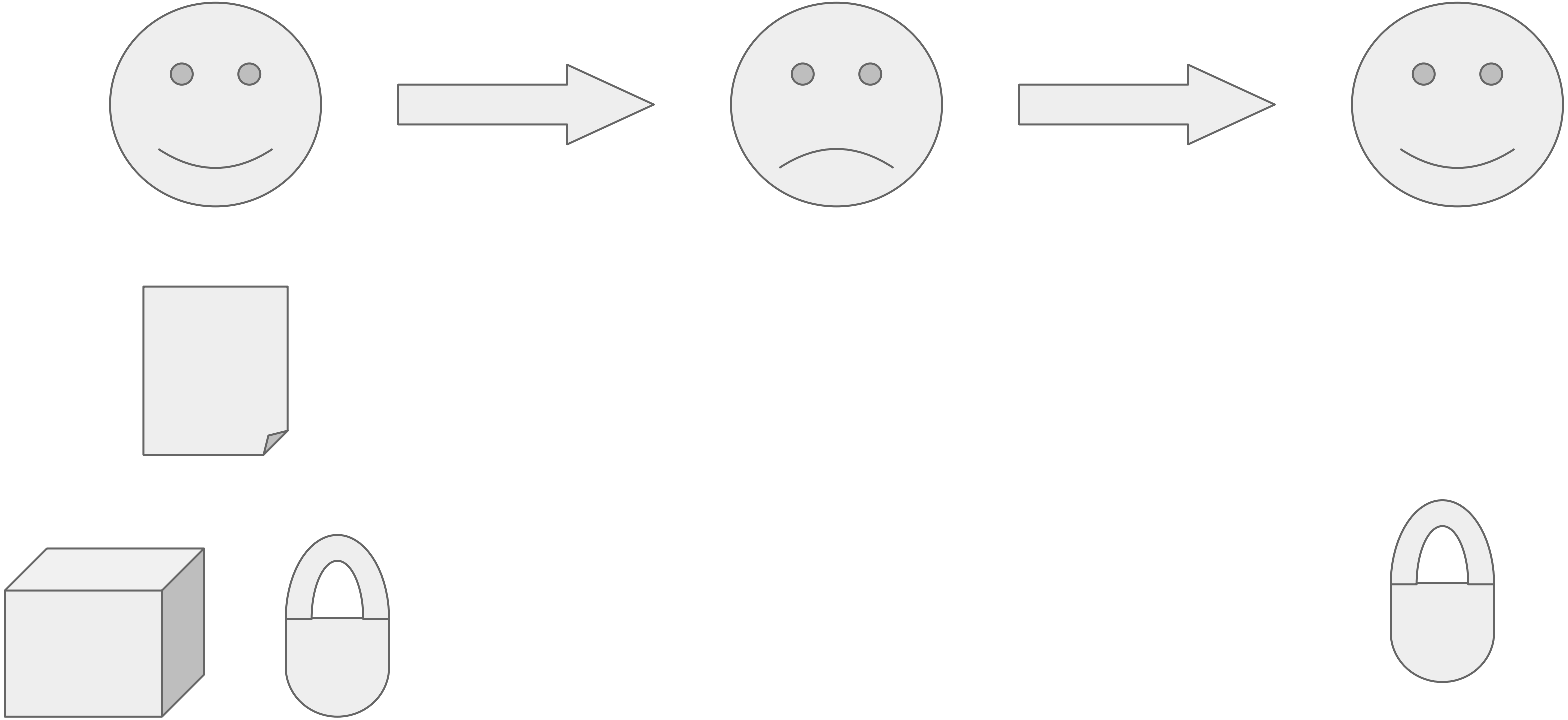

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- A puts the note in the box and locks it. Gives it to B.

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This protocol is secure assuming B does not have any equipment to break the lock.

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Diffie-helman key exchange

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In real-life: F ≡ Multiplying 2 large Prime Numbers

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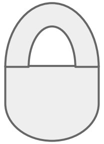

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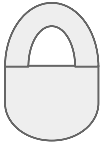

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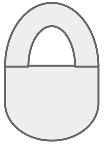
Alter the message to make it look meaningless in such a way that if you have the “key” you can “decrypt”

Diffie-Helman Key Exchange



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BACKUP GAMES

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Ensuring Privacy: Don’t reveal anything more than necessary.

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These can’t be done always.
The question is for what problems can these be done?

Understanding ChatGPT

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Improved:

- Output words depending on their probabilities (along with a other factors).
- Update factors using feedback.

Theoretical Computer Science

- A Problem that needs to be solved.

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Study the amount of resources needed by the model to solve the problem.

Higher Studies in Math/CS theory

Exams and Institutes

Undergraduate Level

- JEE (IISc, IITs & NITs)
- CMI
- ISI
- IAT (IISERs)
- NEST (NISER and CEBS)

Graduate Level (PhD/Int. PhD)

- NET
- GATE
- JEST
- TIFR-GS
- NBHM

TIFR, IISc, CMI, ISI, HRI, NISER, ISI, IISERs, IITs.

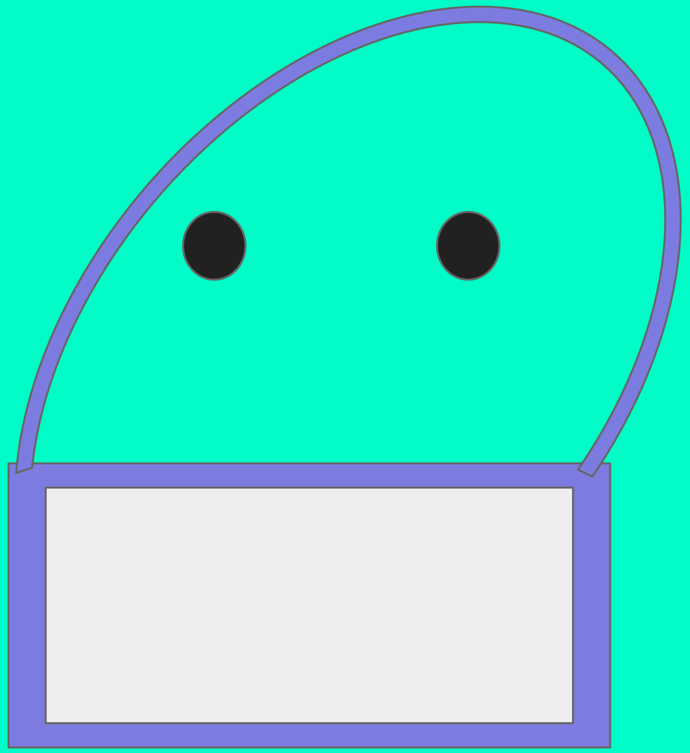
Master's Level

- JAM (for MSc in IISc, IITs & NITs)
- GATE (for MTech in IITs and NITs)

- CMI
- ISI

Just One More Puzzle...

The Picture Hanging Puzzle



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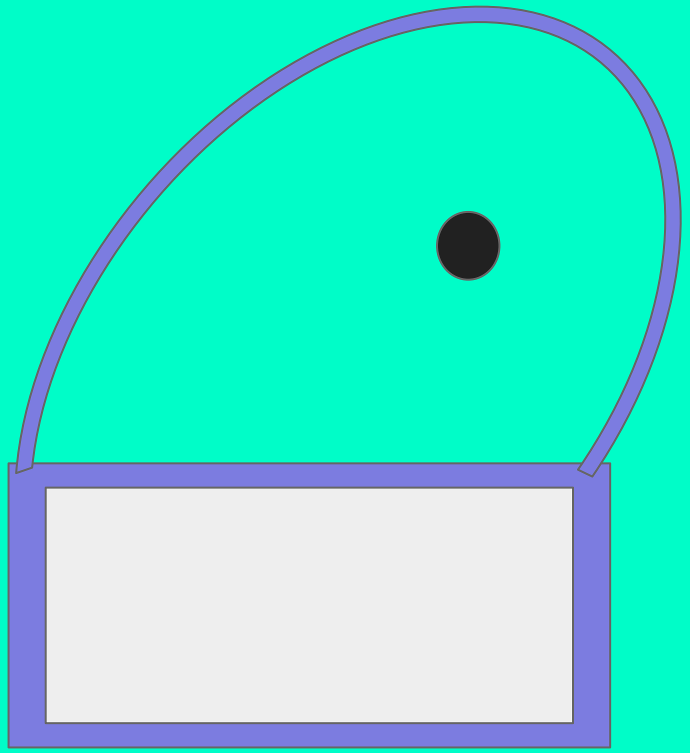

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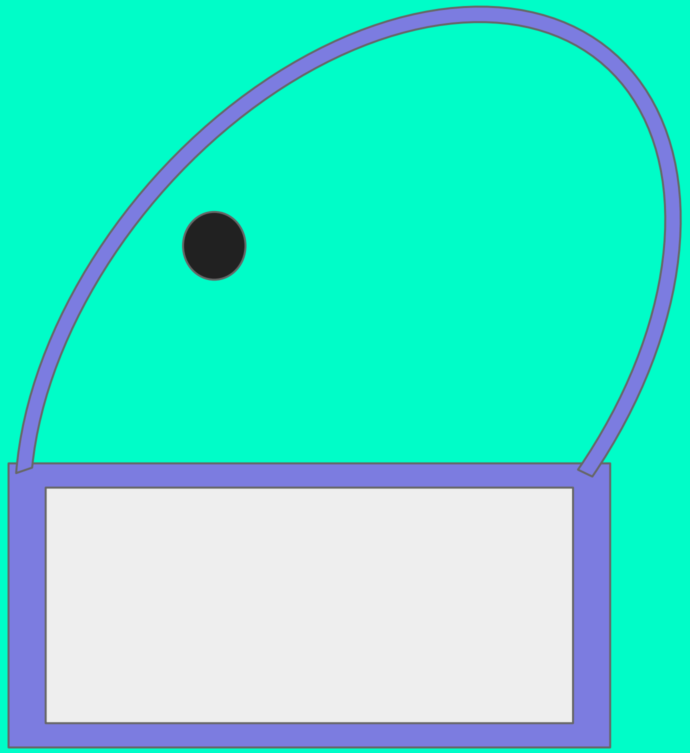

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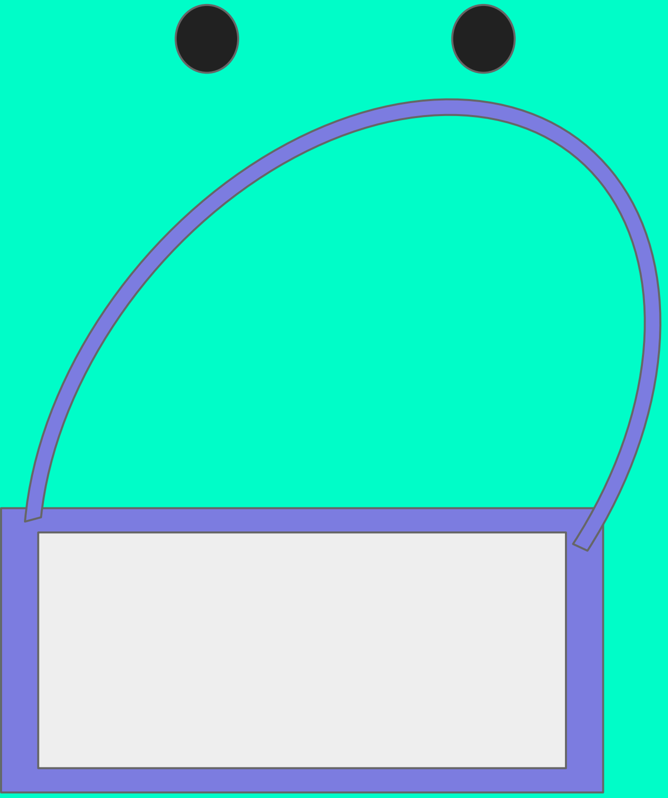

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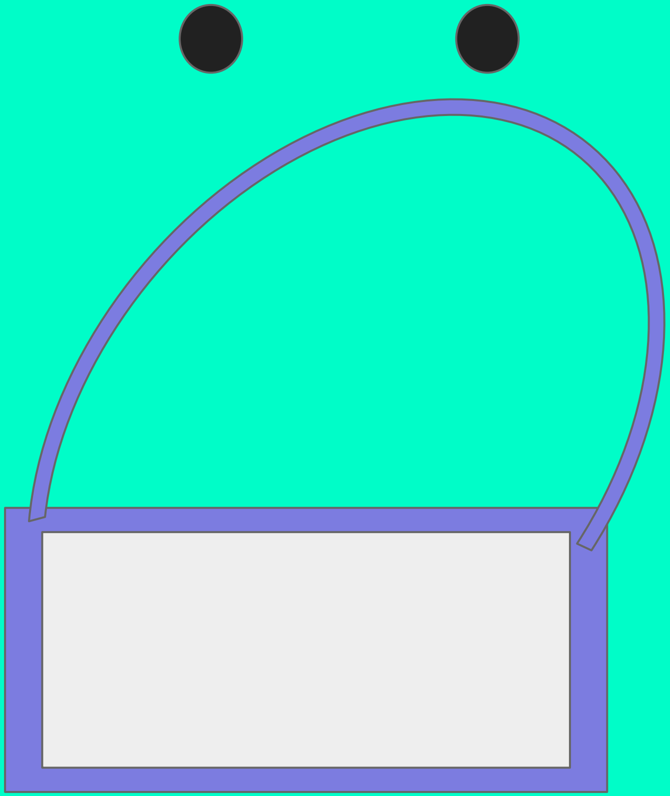

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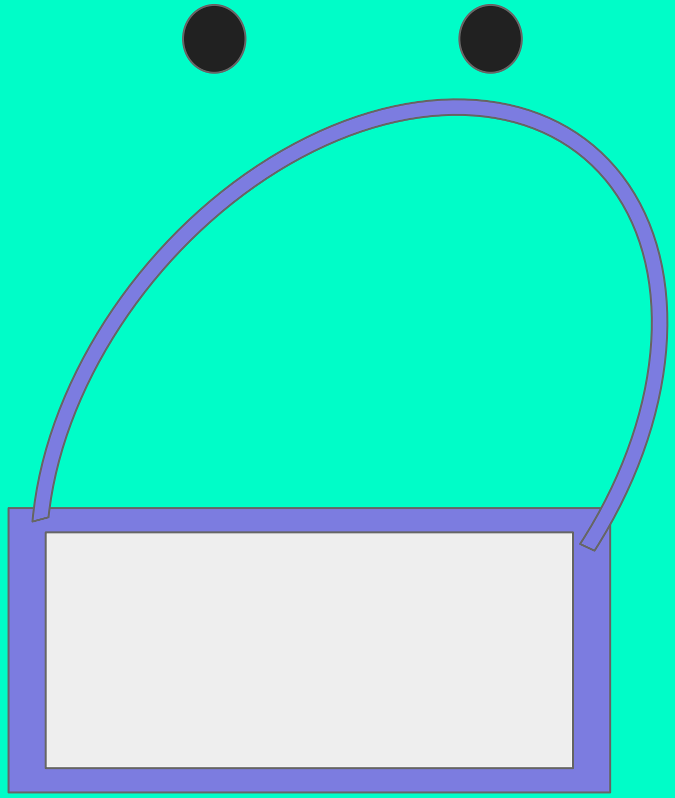
Hang the picture such that:

- If both the pegs are there, then the picture will stay.
- If either one of pegs are taken off, the picture will fall.



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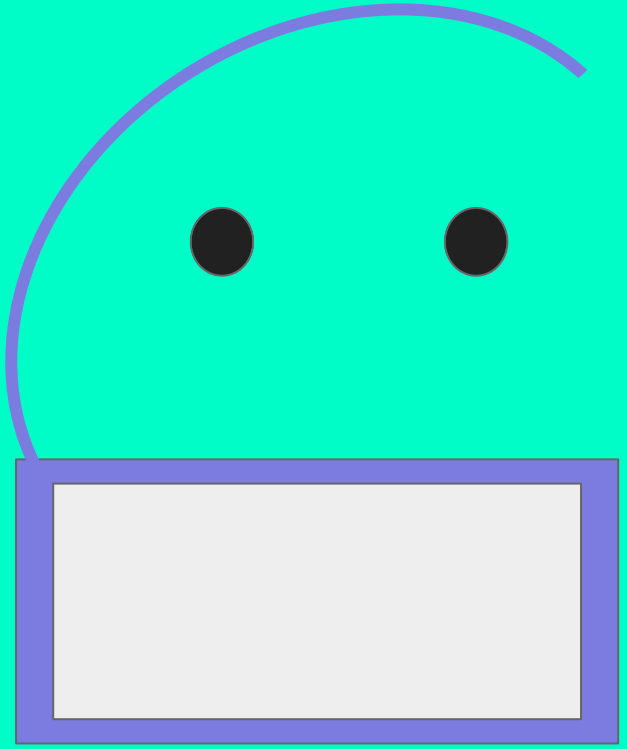
Solution: **A**



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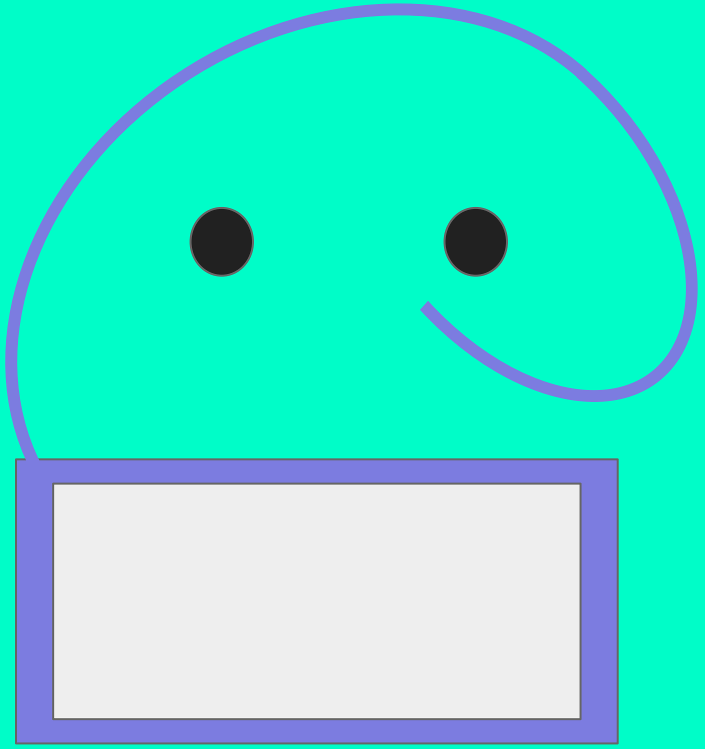
Solution: **AB**



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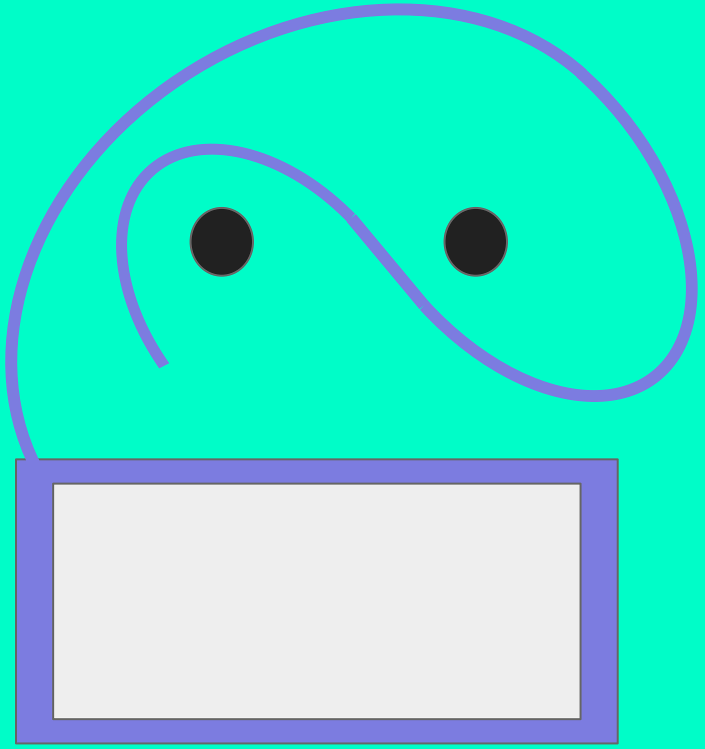

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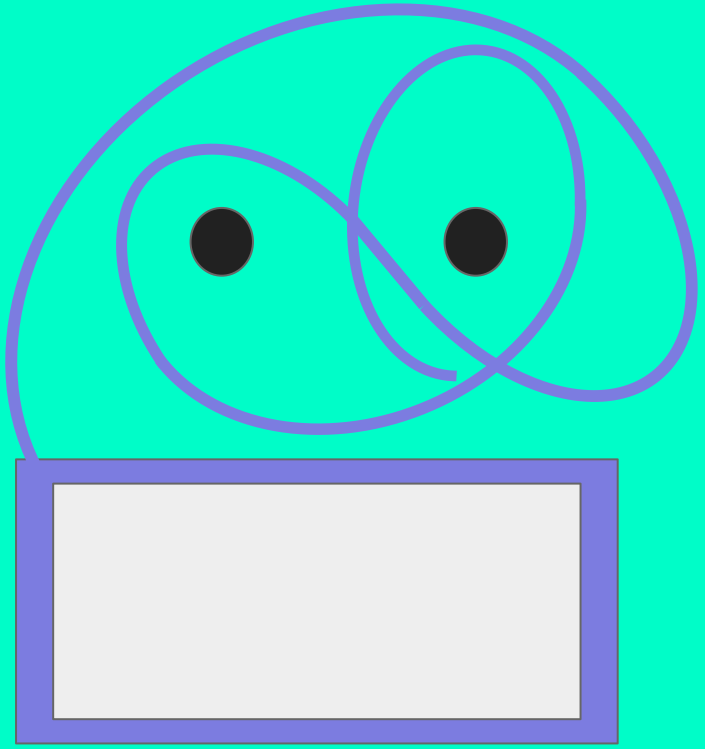

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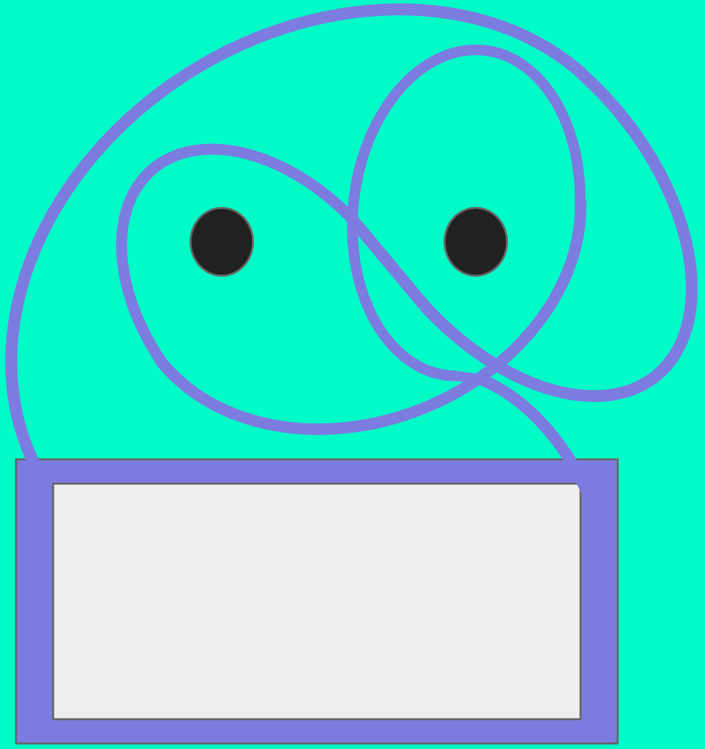

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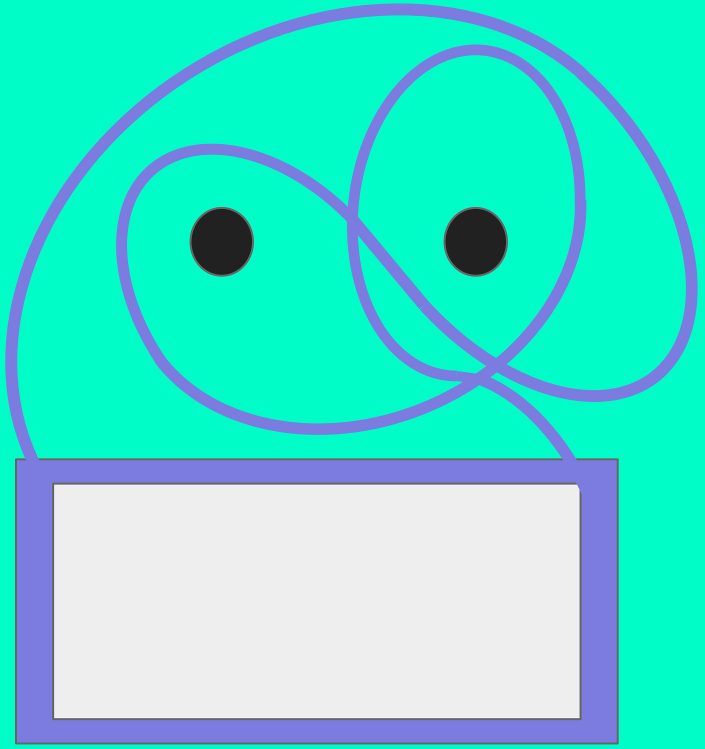
COMMUTATORS



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Check out: Chai and Why?(Dec 20, 2020) by Ramprasad Saptharishi

“Commutators! Hanging pictures and solving Rubik’s Cube”

THANK YOU !

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