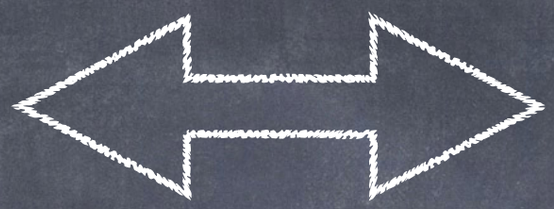
How to convince ourselves that we are NOT stupid...

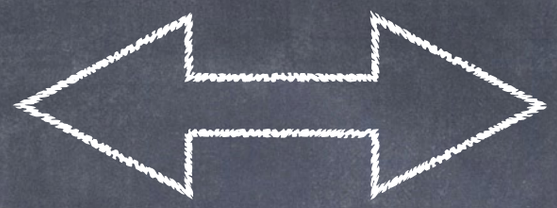
Prerona Chatterjee

STCS @ TIFR

Connecting Cities



Volunteers



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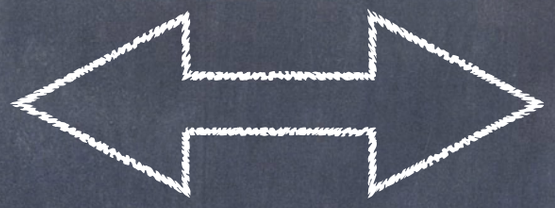
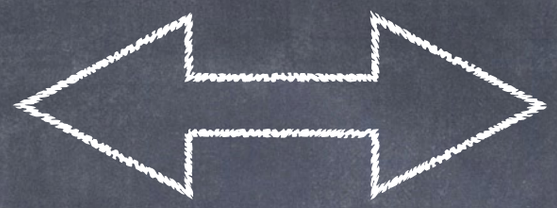


USB Cables



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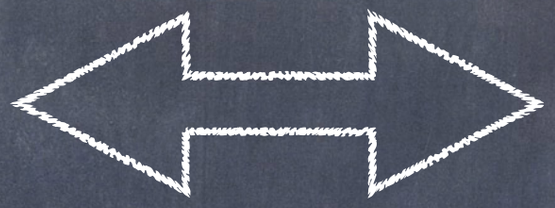
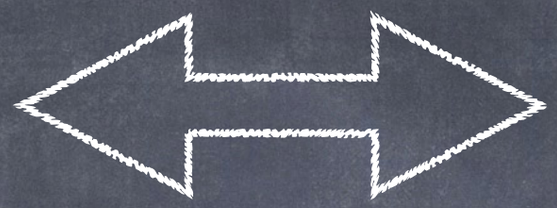


USB Cables

Every pair of cities
should be connected



Volunteers

USB Cables

Every pair of cities
should be connected

Are 5 cables enough?

NO

NO

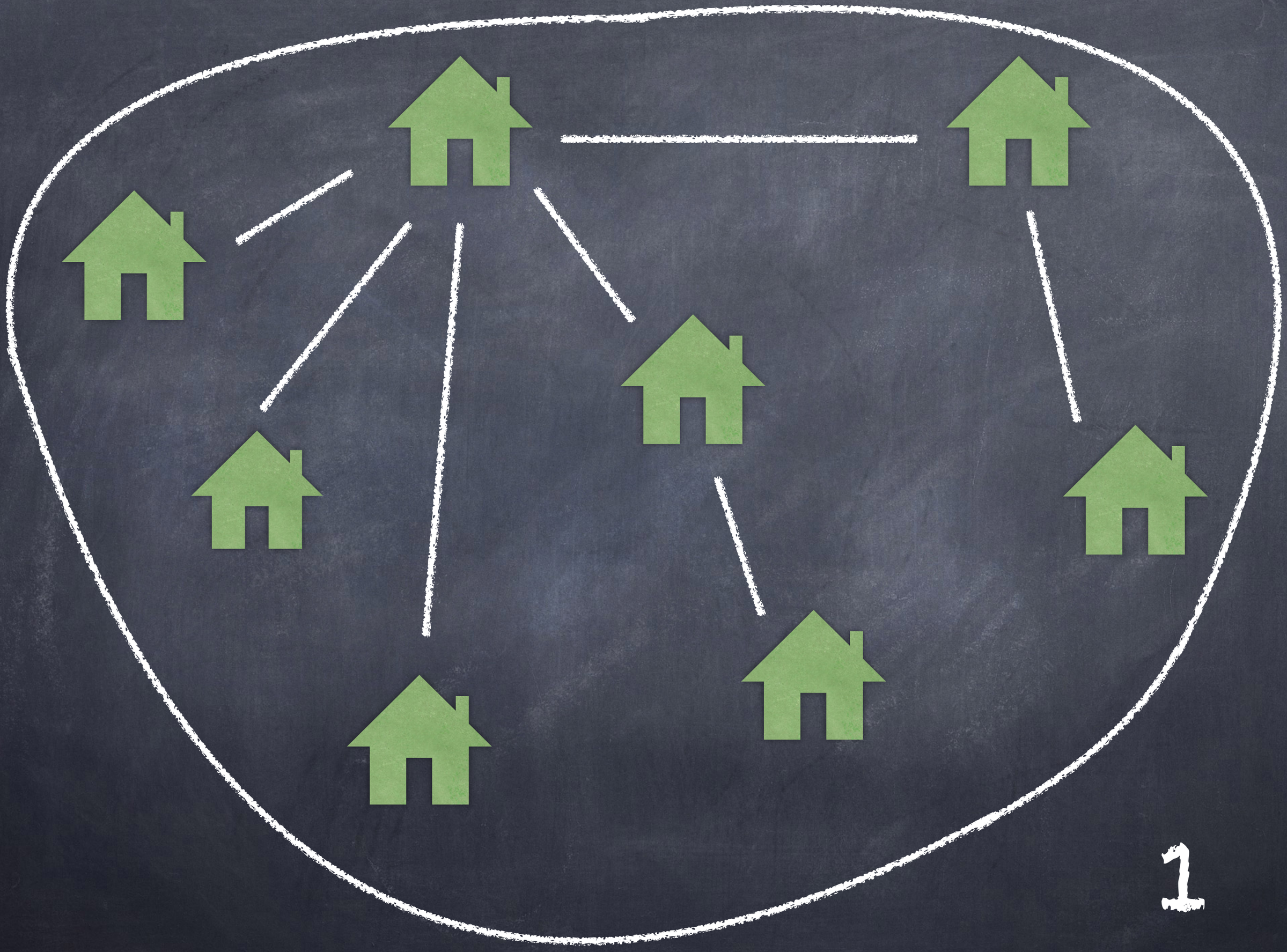
7 cables are required

NO

7 cables are required!

But why?

7



Proof

No. of groups @ the beginning = n

No. of groups in the end = 1

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At each step the no. of groups reduce by at most 1

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No. of groups in the end = 1

At each step the no. of groups reduce by at most 1

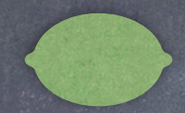
So, the no. of steps reqd. $\geq n-1$

Sorting

1	2	3	4	5	6	7	8	9	10
61	73	38	2	54	89	92	44	13	29

1	2	3	4	5	6	7	8	9	10
61	73	38	2	54	89	92	44	13	29

38 > 13

1	2	3	4	5	6	7	8	9	10
									

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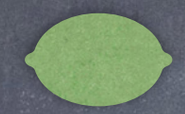
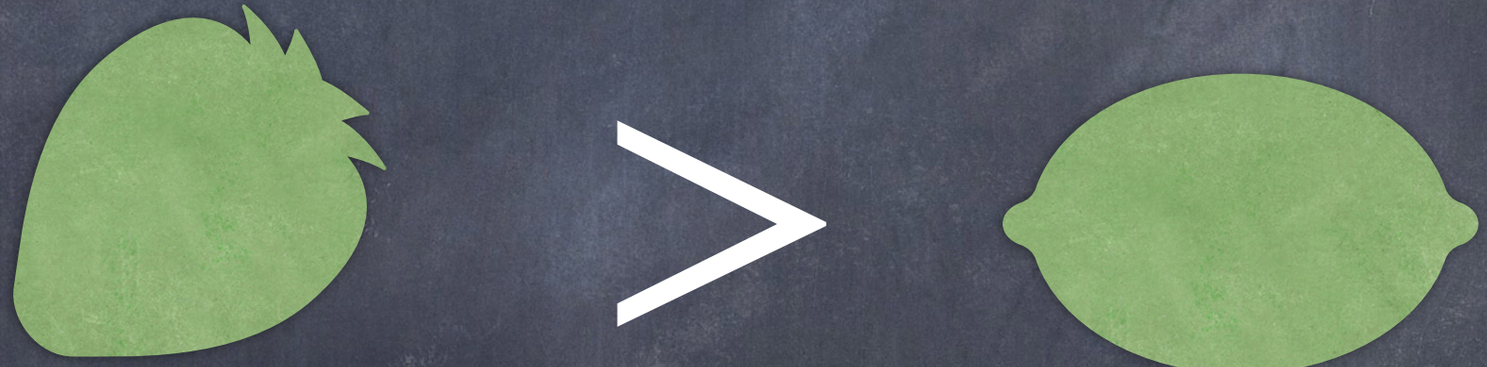
									

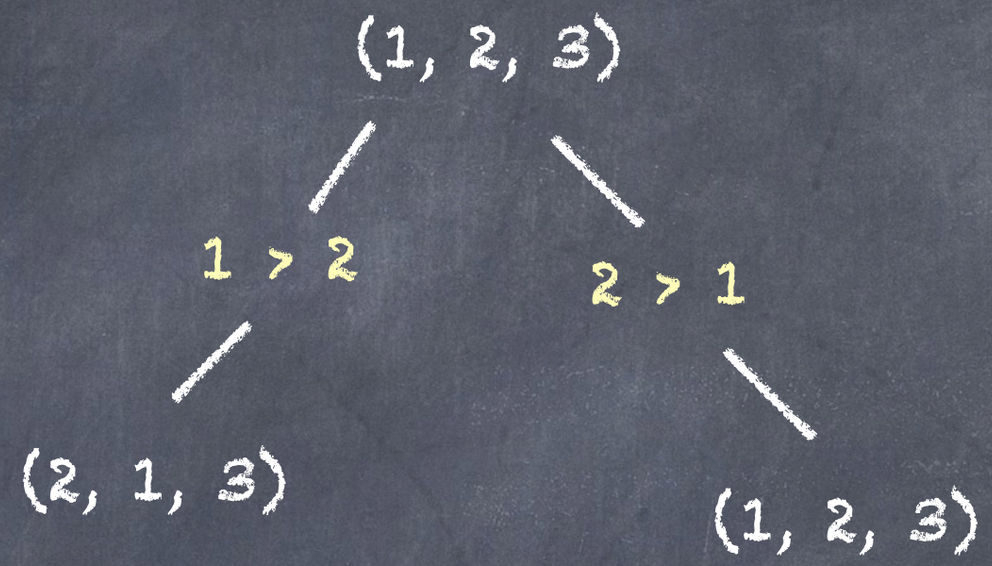


Bubble Sort

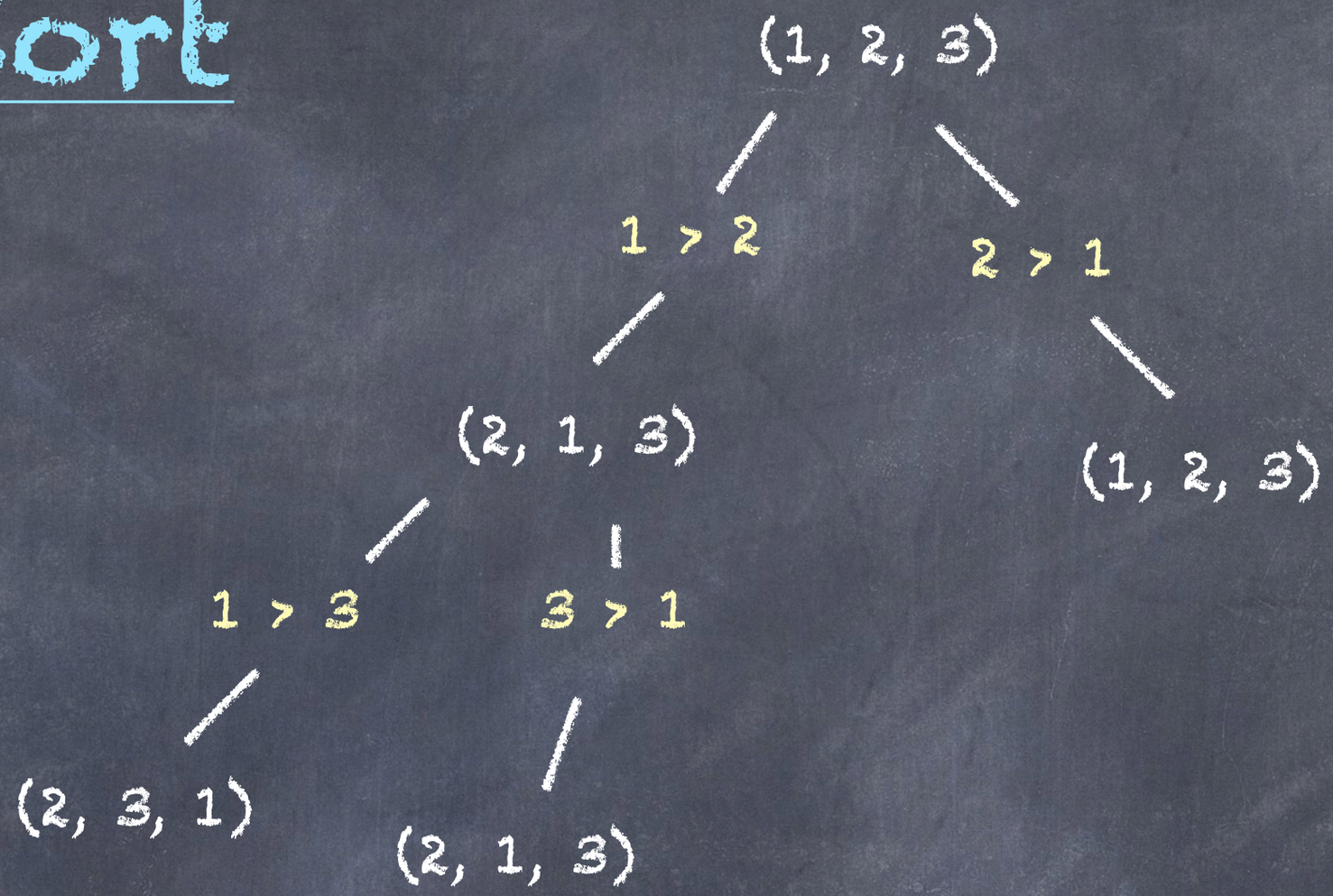
1	2	3
61	73	38

⟺ (1, 2, 3)

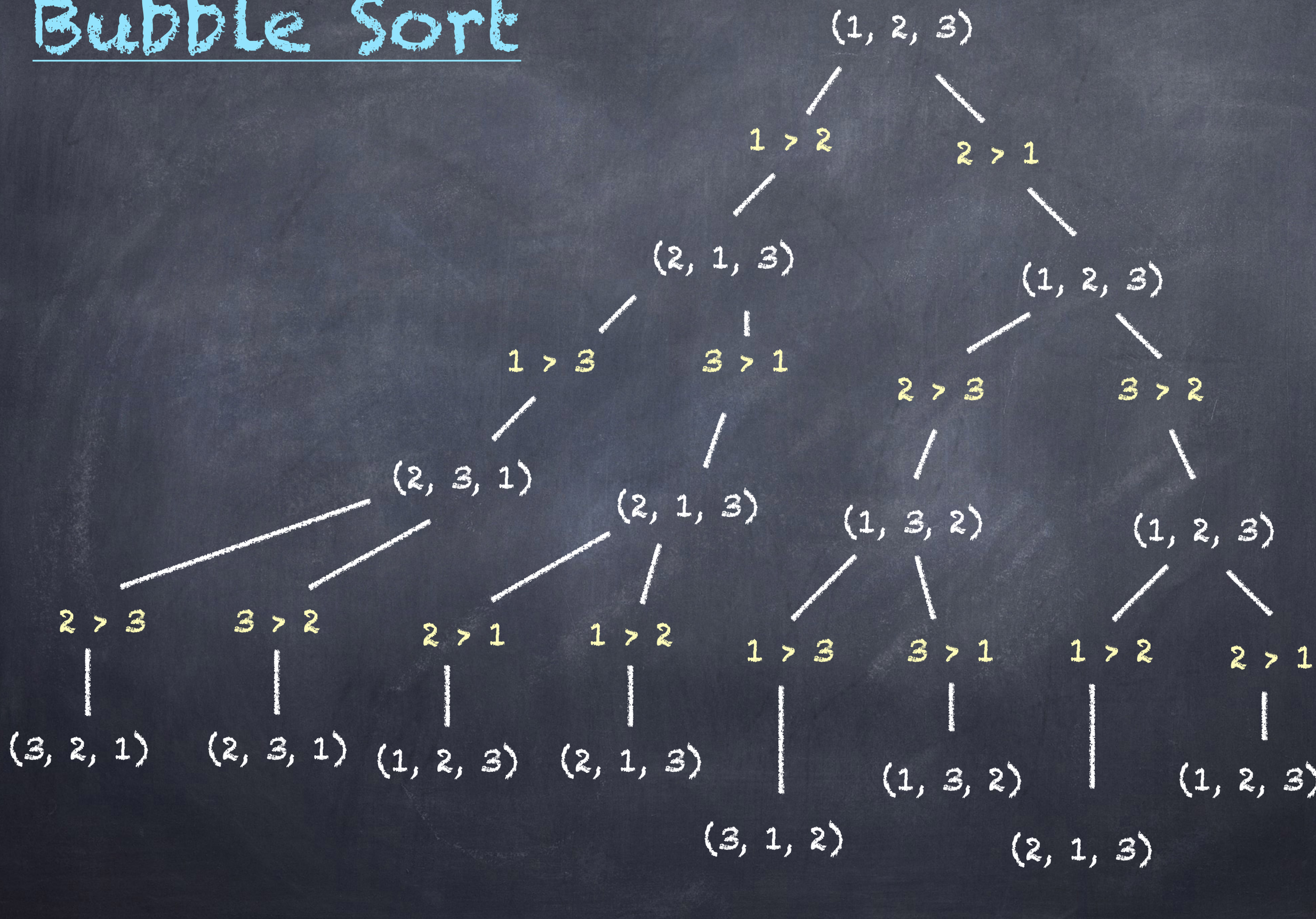
Bubble Sort



Bubble Sort



Bubble Sort


(1, 2, 3)
1 > 2
2 > 1
(2, 1, 3)
(1, 2, 3)
1 > 3
3 > 1
2 > 3
3 > 2
(2, 3, 1)
(2, 1, 3)
(1, 3, 2)
(1, 2, 3)
2 > 3
3 > 2
2 > 1
1 > 2
1 > 3
3 > 1
1 > 2
2 > 1
(3, 2, 1)
(2, 3, 1)
(1, 2, 3)
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Bubble Sort

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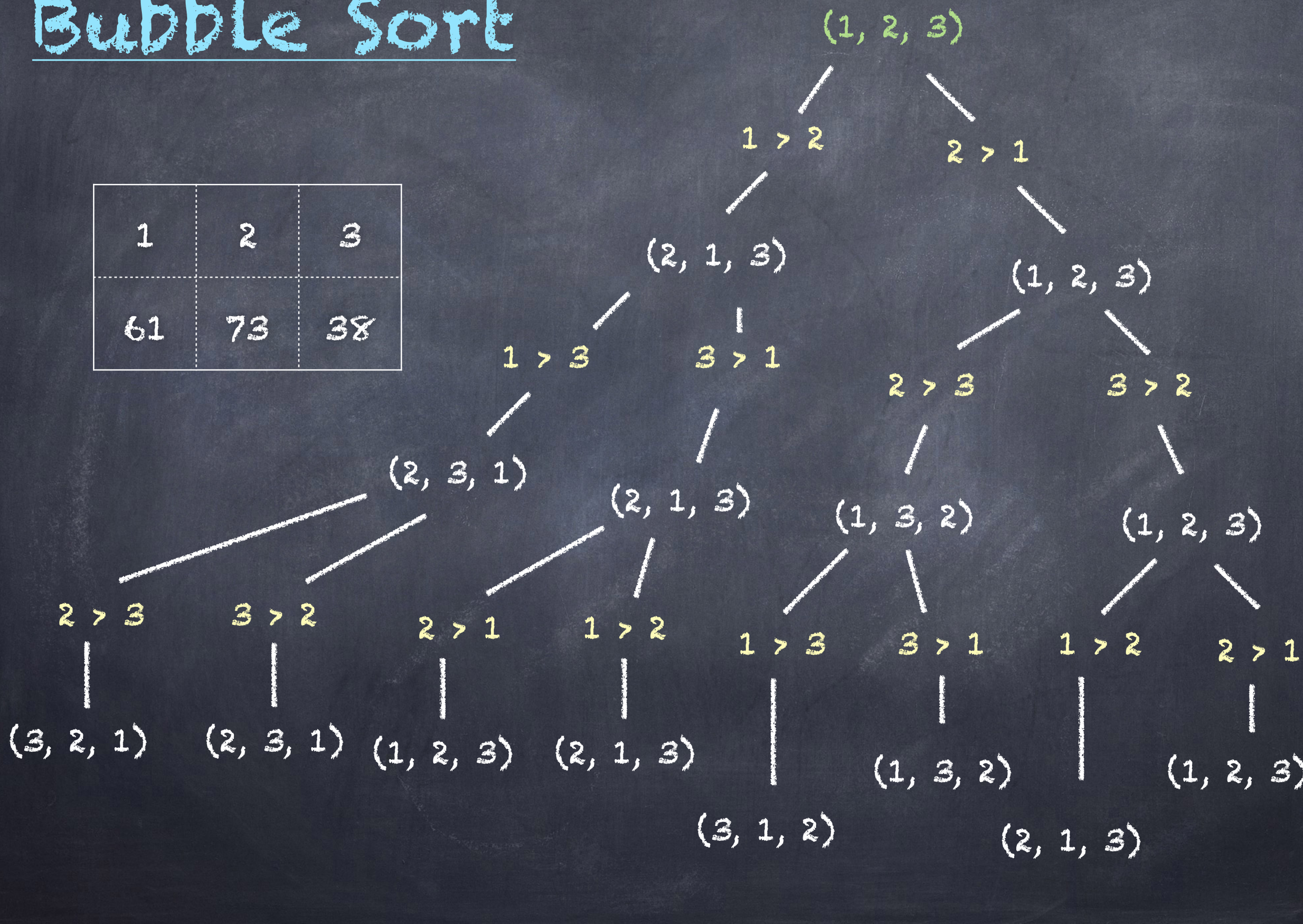

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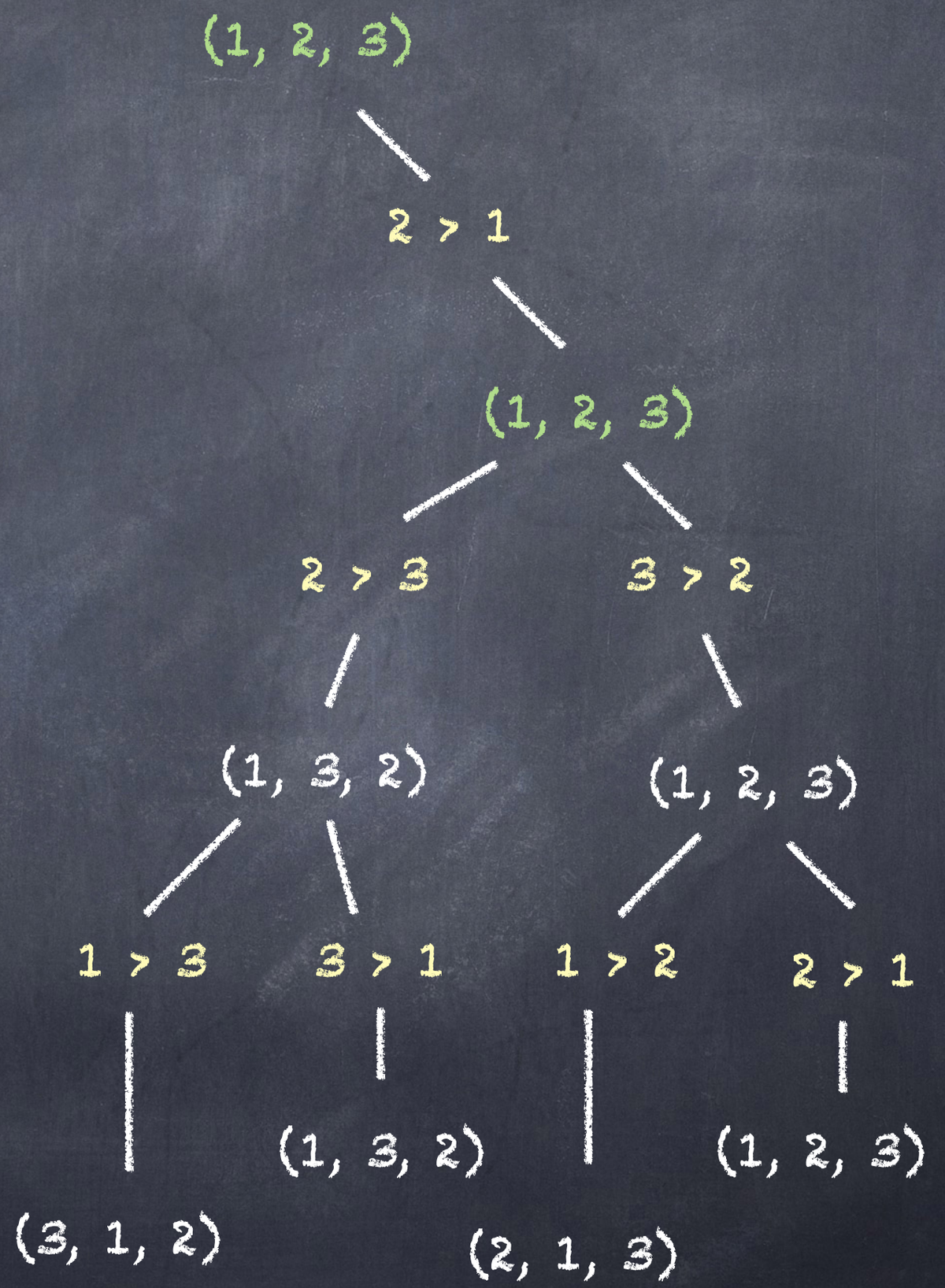
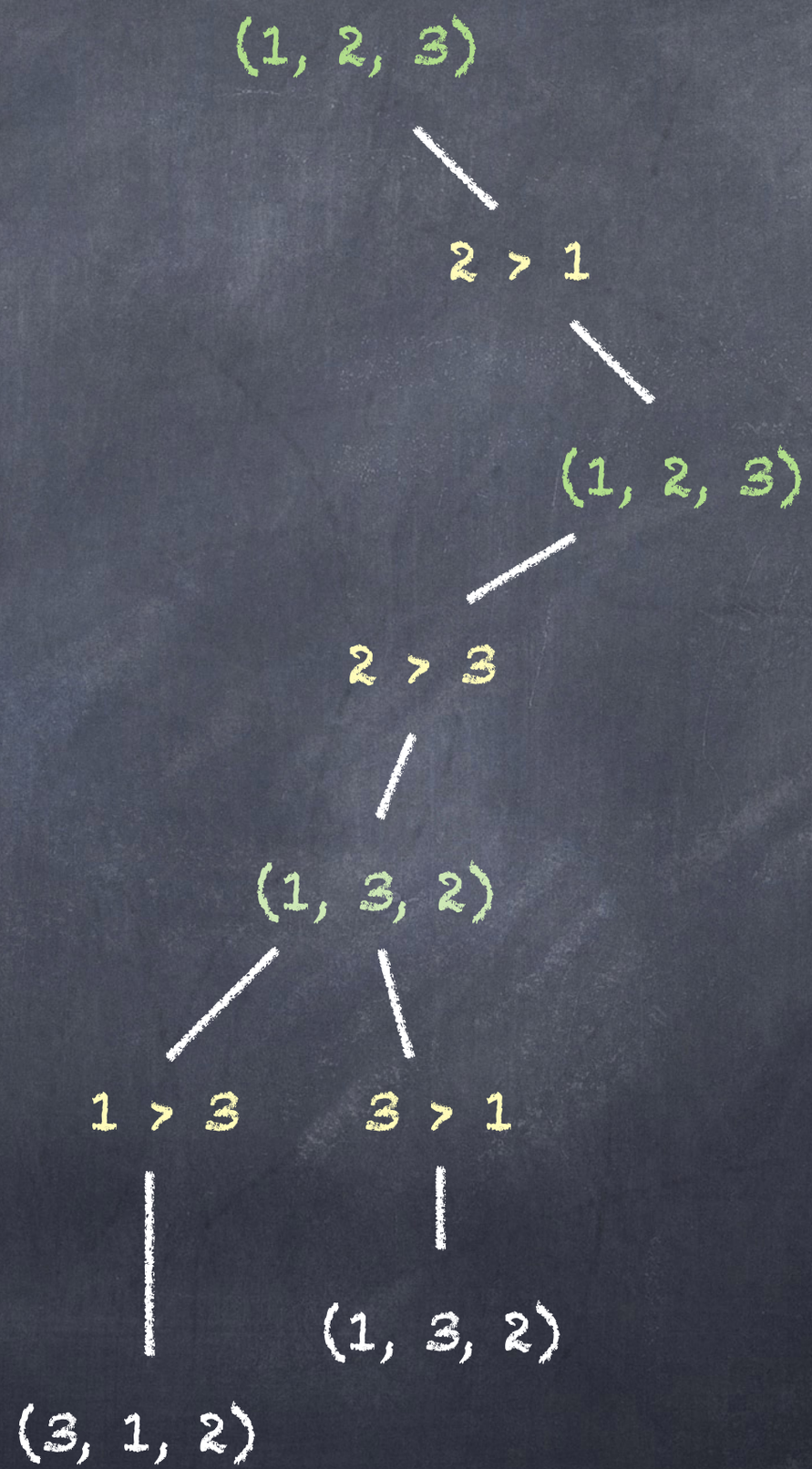


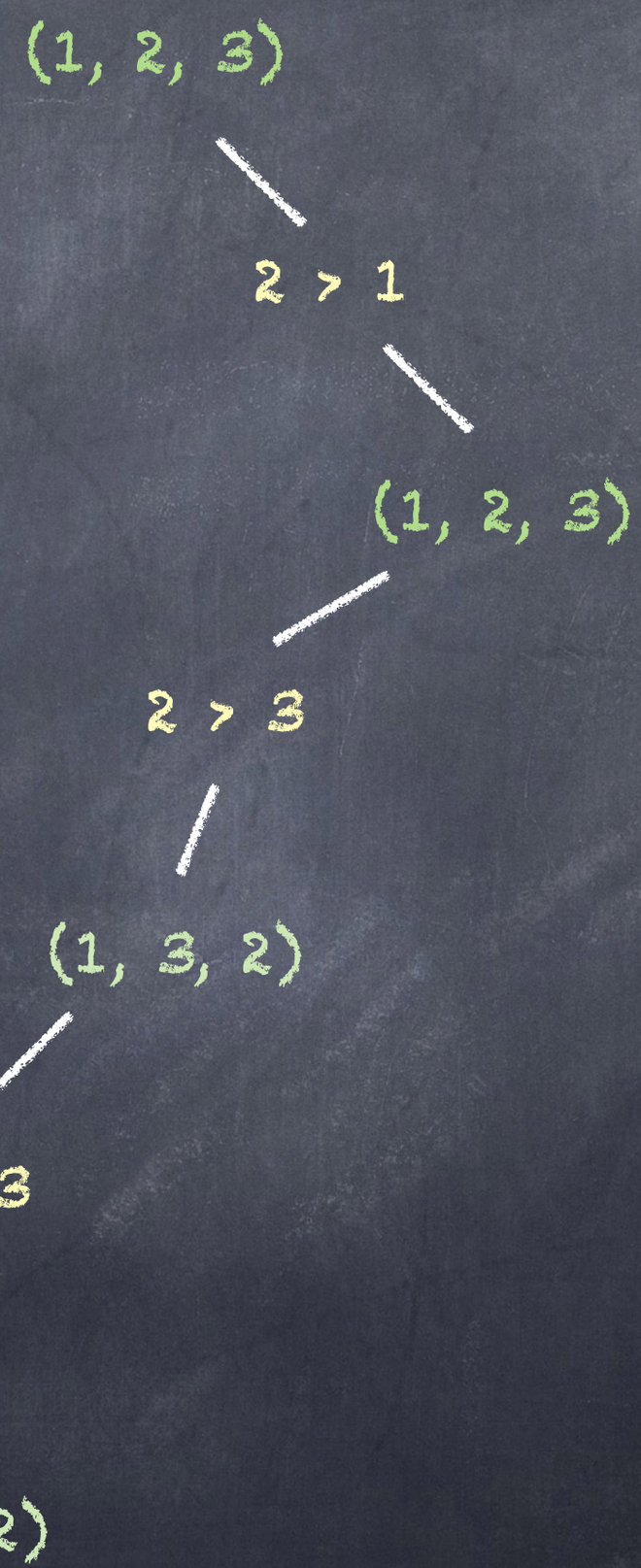
Bubble Sort

1	3	2
61	38	73



Bubble Sort

3	1	2
38	61	73

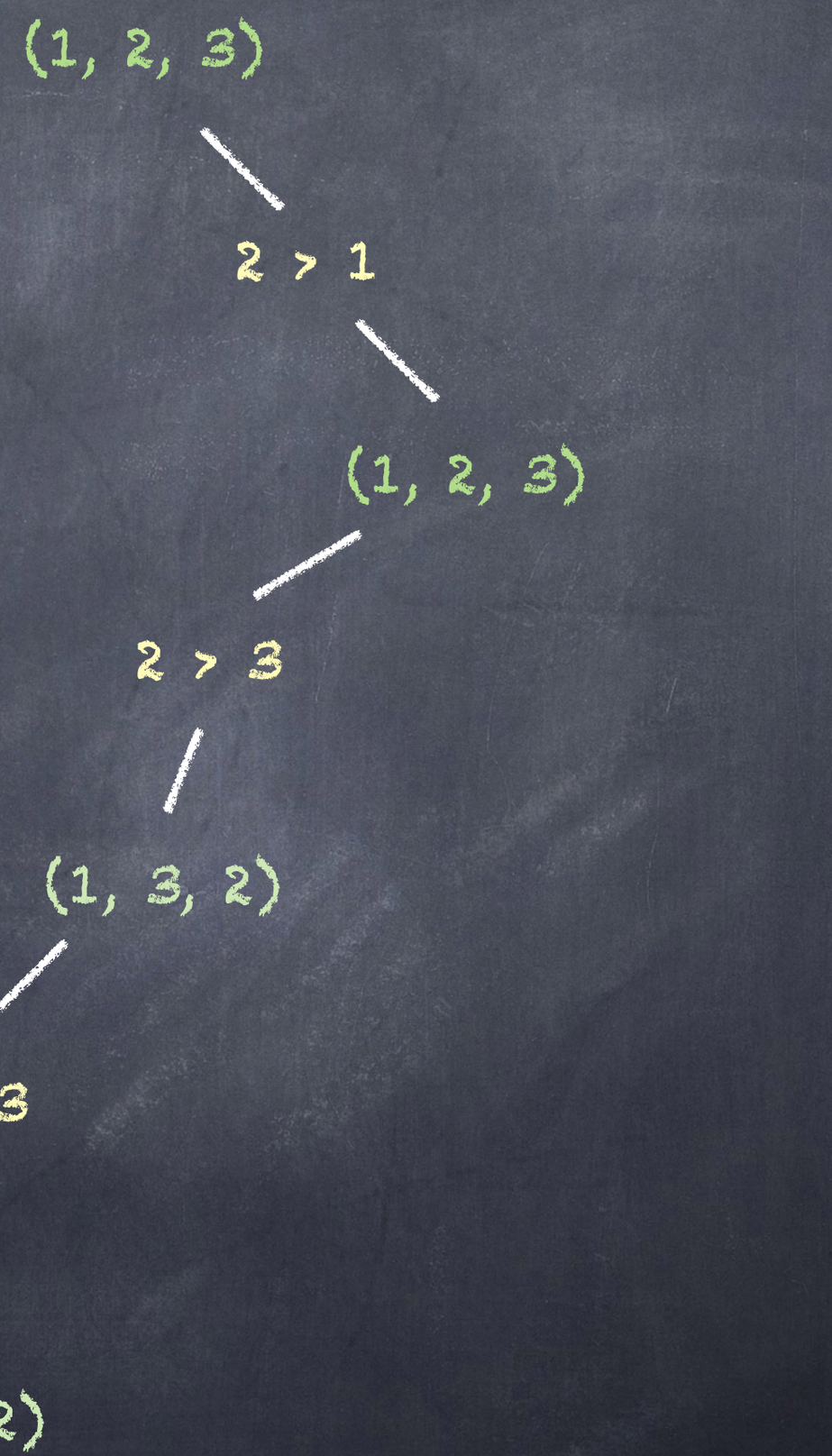


Bubble Sort

3	1	2
38	61	73

#comparisons

= 2 + 1 = 3



Bubble Sort

In general, the running time is:

$$(n-1)+(n-2)+\cdots+1=\frac{n(n-1)}{2}=\Theta(n^2)$$

Algorithms	Running Time
Bubble Sort	$O(n^2)$
Selection Sort	$O(n^2)$
Insertion Sort	$O(n^2)$
Merge Sort	$O(n \log n)$
Quick Sort	$O(n \log n)$
Heap Sort	$O(n \log n)$
.	
.	
.	

Can we do any better?

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- Nothing is known about the kind of data
- Only comparisons allowed

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Nothing better is possible

But why?

A sorting algorithm

Binary tree

A sorting algorithm

Binary tree

Depth

No. of comparisons

A sorting algorithm

Binary tree

Depth

No. of comparisons

Leaf of the binary tree

Final answer

Correct permutation

Proof

Total number of correct permutations possible = n!

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Total number of correct permutations possible = $n!$

$\leq$ the no. of leaves in the tree

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The no. of comparisons required = the depth of the tree

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≤ the no. of leaves in the tree

The no. of comparisons required = the depth of the tree

and this is AT LEAST log (n!) = $\Theta(n \log n)$

Matrix Multiplication

The Question:

How many multiplication operations are required?

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What is currently known:

The trivial algorithm takes $O(n^3)$

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What is currently known:

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Strassen showed it is $O(n^{\log_2 7}) = O(n^{2.807})$

The current best algorithm takes $O(n^{2.3728639})$

The Question:

How many multiplication operations are required?

What is currently known:

The current best upper bound = $O(n^{2.3728639})$

The current best lower bound = $O(n^2)$

Computability
&
Complexity Theory

Problem

Easy?

Hard?

Decision Problems

L : Set of Strings

x : Input string

Question : Does x belong to L?

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Question : Does x belong to L?

Computability:

Is there an algorithm to solve this?

Complexity Theory :

If it does have an algorithm, then how much resource does it require?

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Resources we are interested in:

1. Time required
2. Space required

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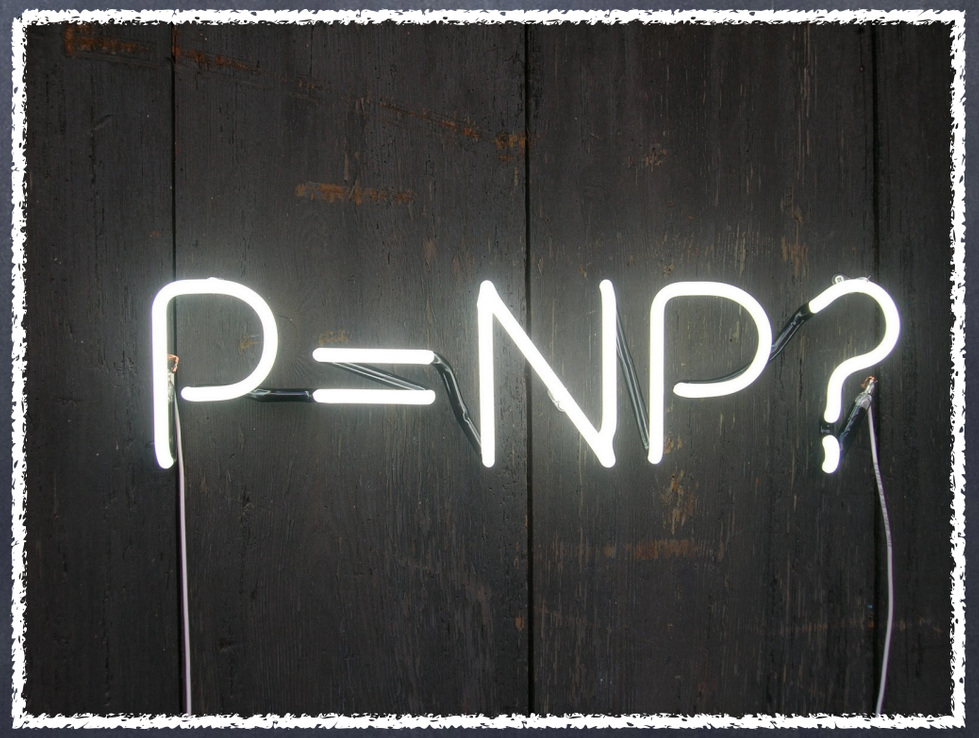

P vs. NP

P: Polynomial time algorithms

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Input Length : n

P: Polynomial time algorithms

Run time: $O(n^k)$ for some const. k

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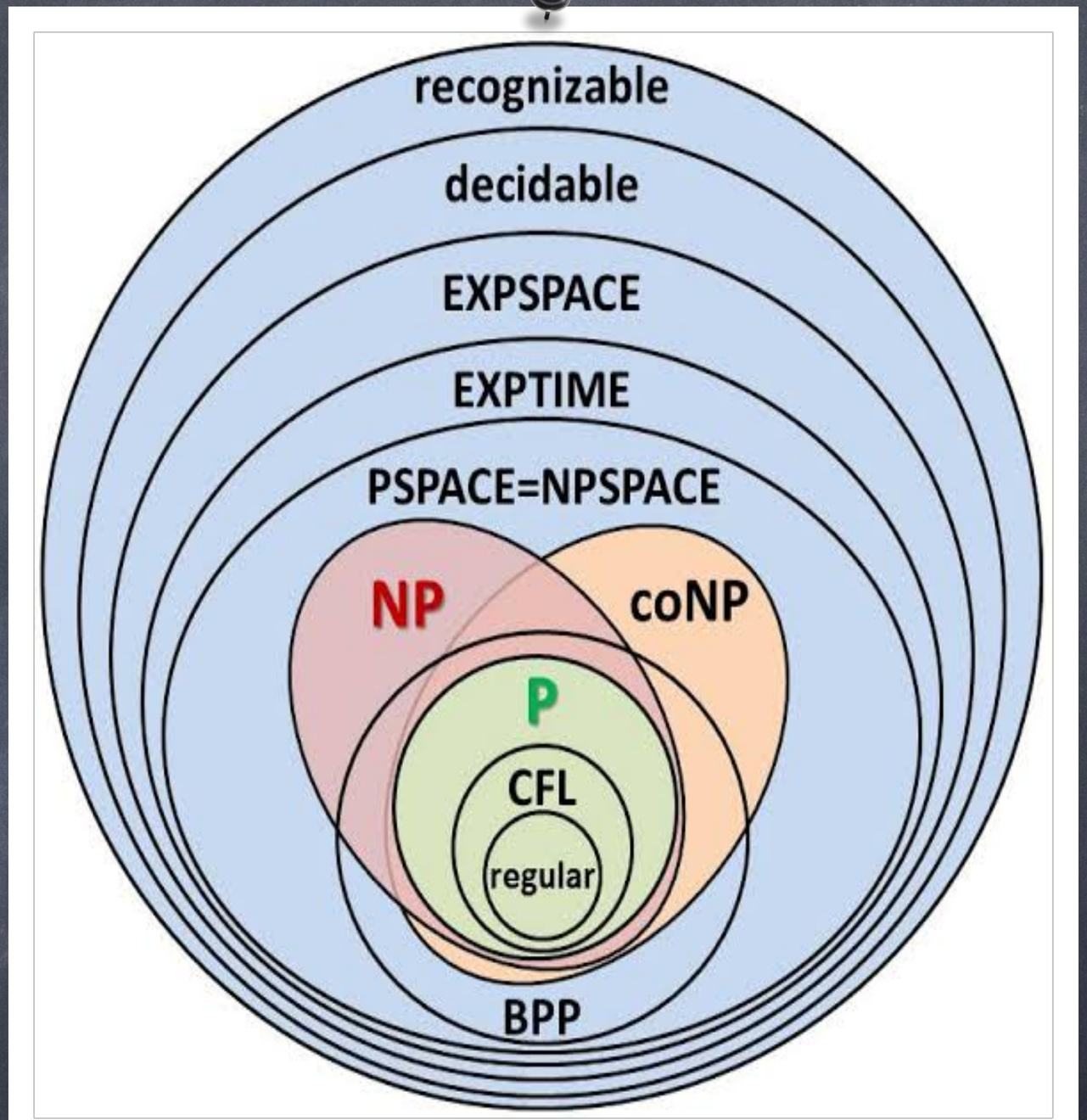
Input Length : n

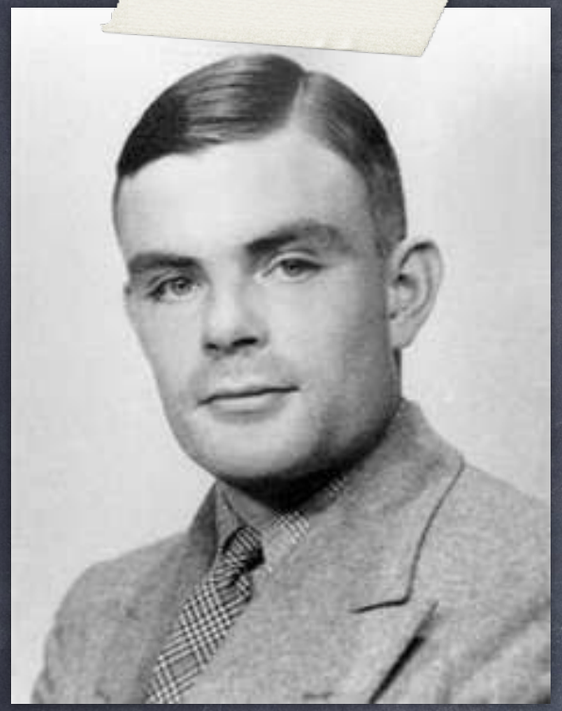
P: Polynomial time algorithms

Run time: $O(n^k)$ for some const. k

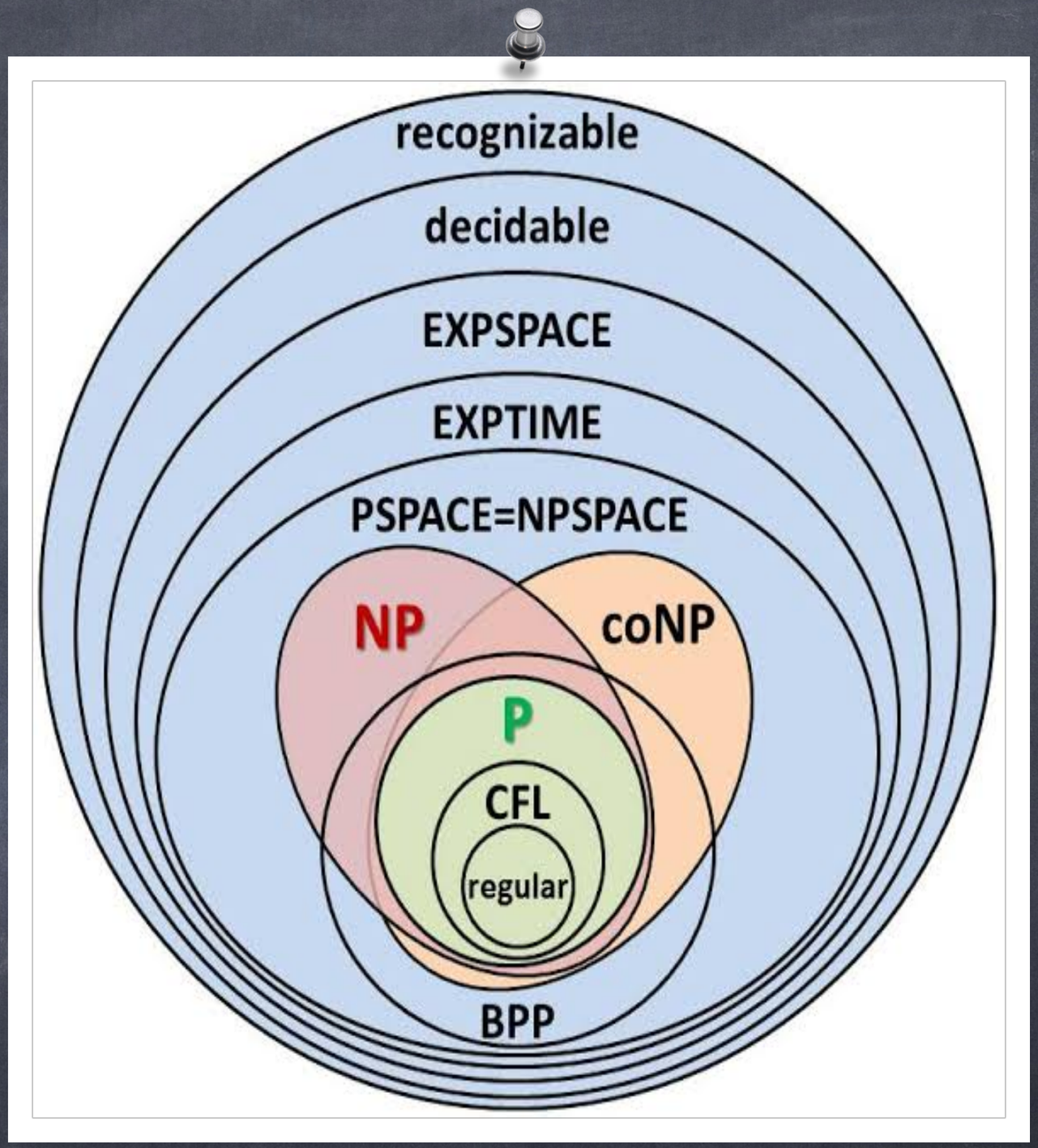
NP: Non-deterministic Polynomial time algorithms

Verifying time: $O(n^k)$ with $O(n^k)$ bits of advice for some const. k


recognizable
decidable
EXPSPACE
EXPTIME
PSPACE=NPSPACE
NP
coNP
P
CFL
regular
BPP



Alan Turing


recognizable
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NP
coNP
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CFL
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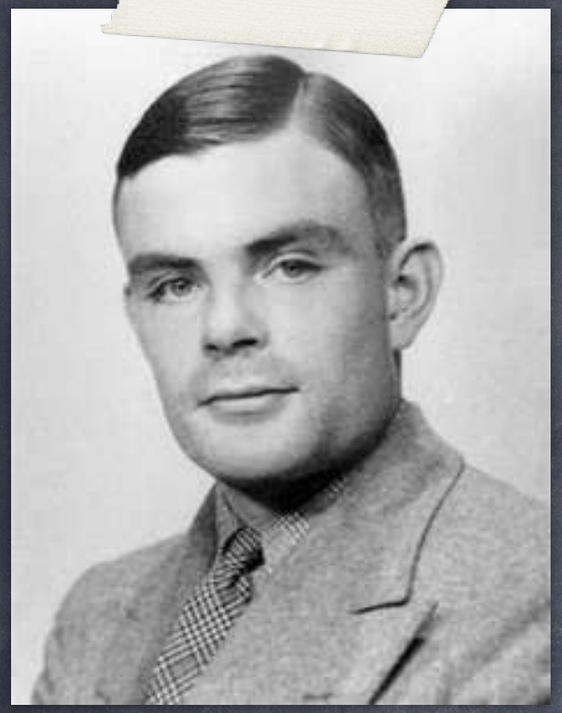

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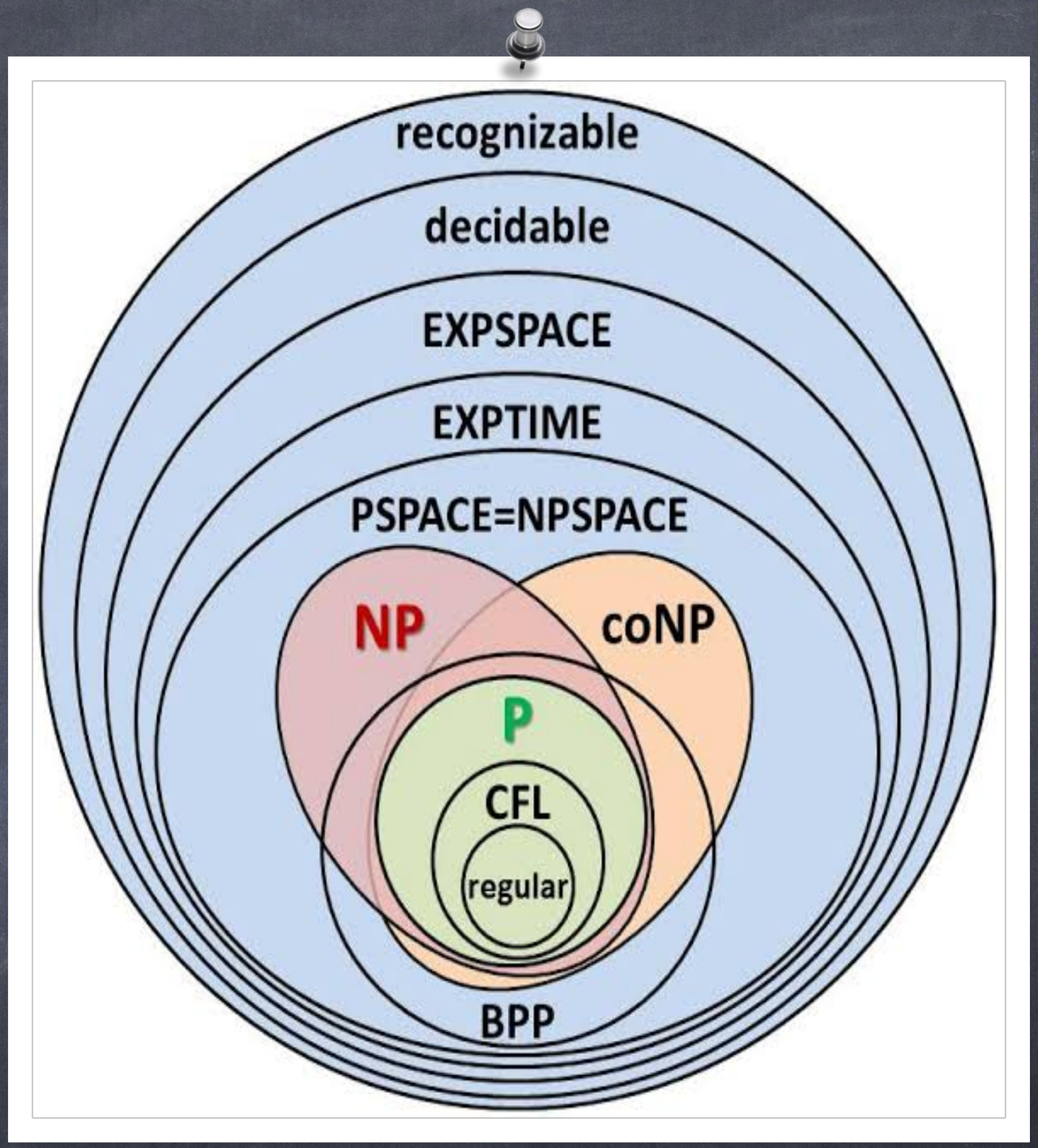
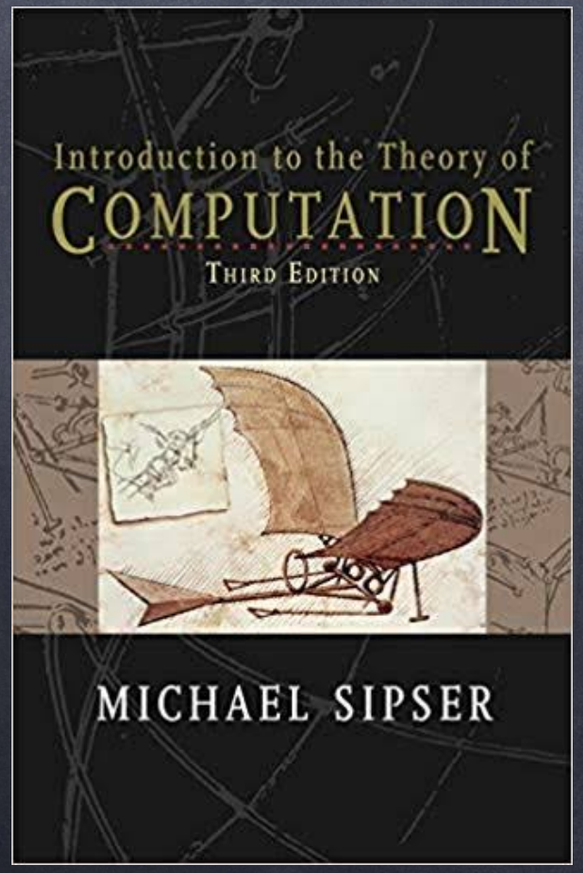


Amazing first book



A bit about what I do...

Algebraic Complexity Theory

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Algebraic Problems

Easy?

Hard?

A bit about what I do...

Algebraic Complexity Theory

Algebraic Problems?

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Algebraic Complexity Theory

Algebraic Problems

1. Primality Test

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1. Primality Test

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1. Primality Test
2. Matrix Multiplication

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Algebraic Problems

1. Primality Test
2. Matrix Multiplication
3. Integer Factorisation

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Algebraic Complexity Theory

Algebraic Problems

1. Primality Test
2. Matrix Multiplication
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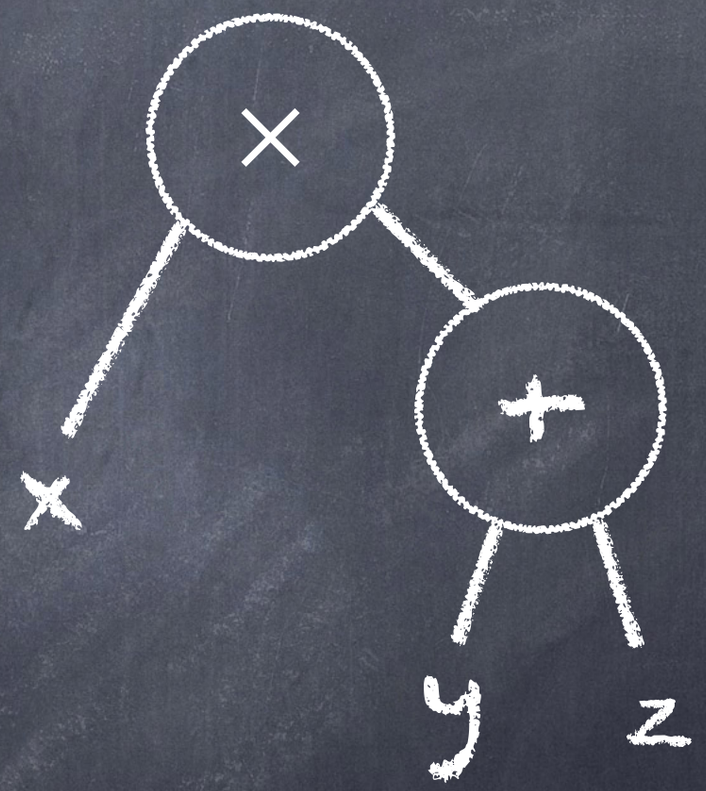
4. Algebraic Circuit Complexity



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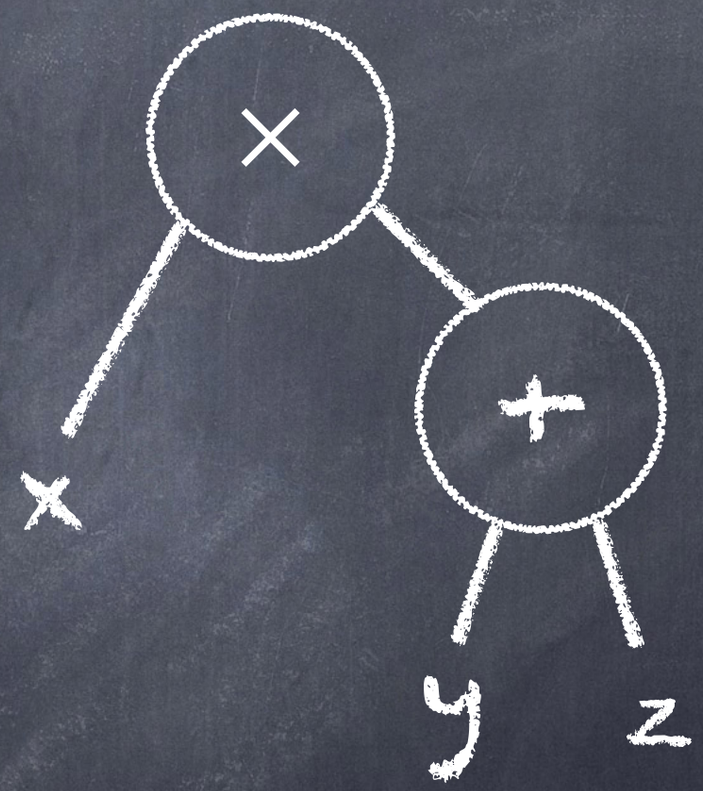

and many more....