Alice, Bob, and some bags of cash

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Alice and Bob play a game

Alice and Bob play a game



Alice and Bob play a game

Game Play

- Alice and Bob go into separate rooms
- They each flip a coin and note down the result.
- Alice writes down a guess as to the result of Bob's coin flip, and Bob writes down a guess as to Alice's flip.

Alice and Bob play a game

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If both guesses are wrong then they both lose. Other wise they both win.

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Alice and Bob are teammates - they will win or lose the cash together.

If both guesses are wrong then they both lose. Other wise they both win.

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Alice and Bob play a game

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Alice flips a coin

Guesses result of
Bob’s coin flip.

Alice and Bob play a game

Alice flips a coin

Guesses result of
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Bob flips a coin

Guesses result of
Alice’s coin flip.

Alice and Bob play a game

Alice flips a coin

Guesses result of Bob's coin flip.

Bob flips a coin

Guesses result of Alice's coin flip.

They win if at least one of them get it right.

Alice and Bob play a game

Alice flips a coin

Guesses result of
Bob's coin flip.

Bob flips a coin

Guesses result of
Alice's coin flip.

They win if at least one of them get it right.
What should they guess?

Alice and Bob play a game

Alice’s Toss	Bob’s Toss	Guesses that lead to Loss
Head	Head	Alice -> Tail & Bob -> Tail
Head	Tail	Alice -> Head & Bob -> Tail
Tail	Head	Alice -> Tail & Bob -> Head
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Case 1: Alice & Bob have same result

Case 2: Alice & Bob have different results

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Winning Strategy

Alice guesses whatever her result is.

Bob guesses the opposite of what his result is.

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Alice will assume this is the case.

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Alice will assume this is the case.

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Alice and Bob play another game

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Game Play

- Alice and Bob go into separate rooms

Alice and Bob play another game

Game Play

- Alice and Bob go into separate rooms
- Charlie goes into Alice’s room and flips a coin. Let the result be X.

Alice and Bob play another game

Game Play

- Alice and Bob go into separate rooms
- Charlie goes into Alice's room and flips a coin. Let the result be X. He then asks Alice to choose either to keep the cash or not.

Alice and Bob play another game

Game Play

- Alice and Bob go into separate rooms
- Charlie goes into Alice's room and flips a coin. Let the result be X. He then asks Alice to choose either to keep the cash or not.
- Charlie goes into Bob's room and flips a coin. Let the result be Y. He then asks Bob to choose either to keep the cash or not.

Alice and Bob play another game

Game Play

- Alice and Bob go into separate rooms
- Charlie goes into Alice's room and flips a coin. Let the result be X. He then asks Alice to choose either to keep the cash or not.
- Charlie goes into Bob's room and flips a coin. Let the result be Y. He then asks Bob to choose either to keep the cash or not.

If X=Y=Head, they both win if and only if they make opposite choices.

Alice and Bob play another game

Game Play

- Alice and Bob go into separate rooms
- Charlie goes into Alice's room and flips a coin. Let the result be X. He then asks Alice to choose either to keep the cash or not.
- Charlie goes into Bob's room and flips a coin. Let the result be Y. He then asks Bob to choose either to keep the cash or not.

If X=Y=Head, they both win if and only if they make opposite choices.

Otherwise, they both win if and only if they make the same choice.

Alice and Bob play another game

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- Alice and Bob go into separate rooms
- Charlie goes into Alice's room and flips a coin. Let the result be X. He then asks Alice to choose either to keep the cash or not.
- Charlie goes into Bob's room and flips a coin. Let the result be Y. He then asks Bob to choose either to keep the cash or not.

Alice and Bob are teammates - they will win or lose the cash together. Before the game starts, they can talk to each other and agree on a strategy.

Alice and Bob play another game

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
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- Alice and Bob go into separate rooms
- Charlie goes into Alice's room and flips a coin. Let the result be X. He then asks Alice to choose either to keep the cash or not.
- Charlie goes into Bob's room and flips a coin. Let the result be Y. He then asks Bob to choose either to keep the cash or not.

Alice and Bob play another game






Alice and Bob play another game

Charlie flips a coin

Asks Alice if she wants
to keep the cash.

Alice and Bob play another game

Charlie flips a coin

Asks Alice if she wants to keep the cash.

Charlie flips a coin

Asks Bob if he wants to keep the cash.

Alice and Bob play another game



Charlie flips a coin

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Charlie flips a coin

Asks Bob if he wants
to keep the cash.

What should Alice and Bob have said?

Alice and Bob play another game

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ALICE AND BOB PLAY ANOTHER GAME

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Alice and Bob play another game

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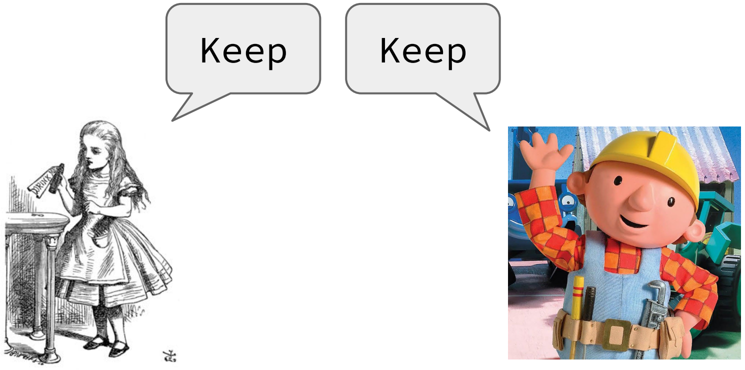

Alice and Bob win with probability 75%

Alice and Bob play another game

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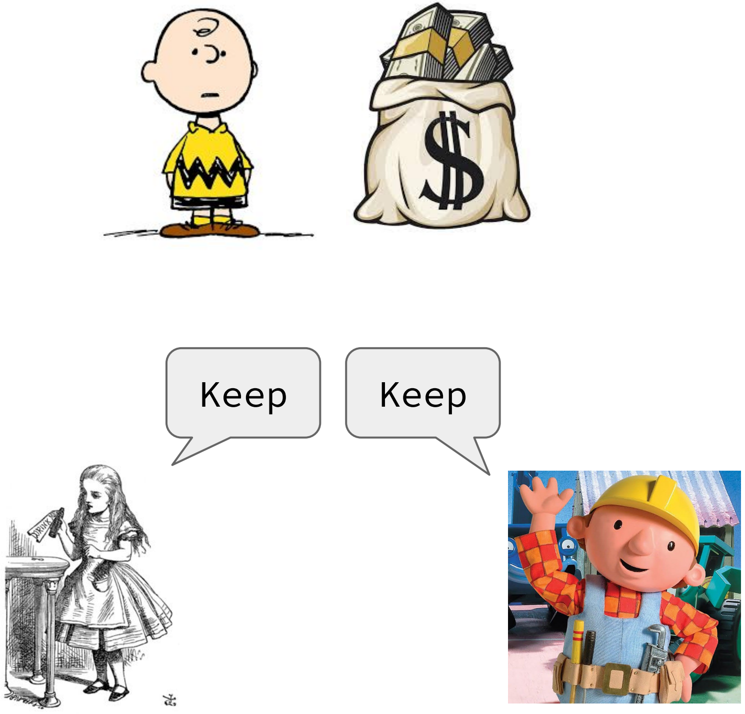


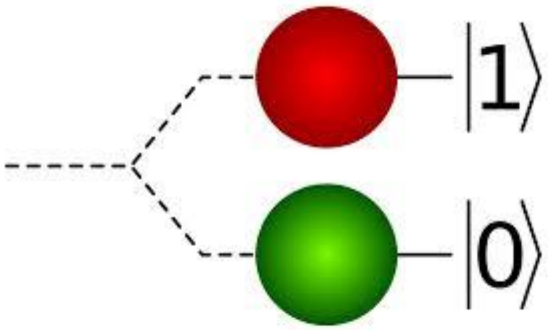
Bell's Inequality:
Alice and Bob can not come up with a better strategy even if they had access to shared random bits.

Alice and Bob win with probability 75%

What if they had access to shared quantum bits?

Quantum Bits (Qubits)

- A new data type with two basic values: |0> and |1>



Quantum Bits (Qubits)

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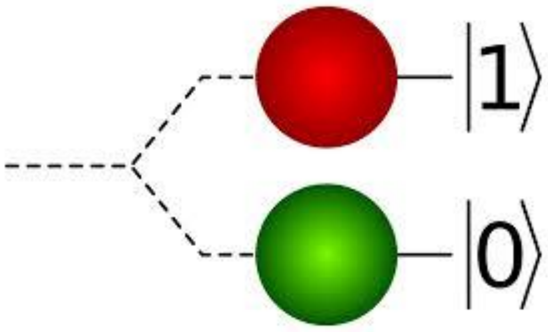
- Magic of Quantum: A quantum bit can be both |0> and |1> until a measurement is made.



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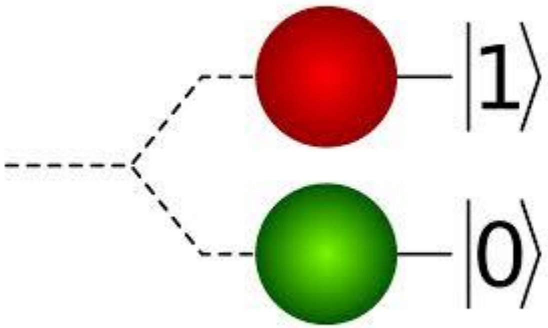
- A qubit has the form a|0> + b|1> for a,b ∊ [0,1] such that

$$a^2 + b^2 = 1.$$



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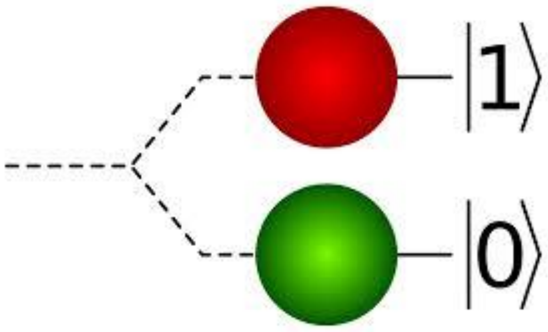
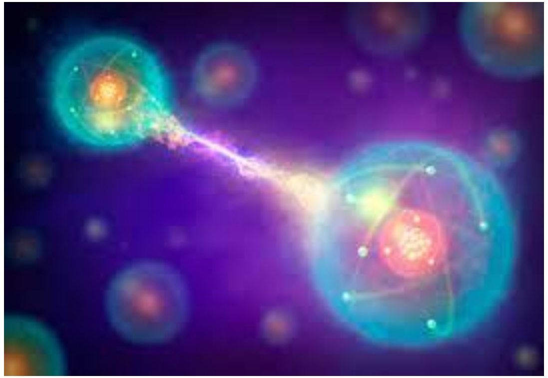


When such a qubit is measured, the probability of measuring |0> is a^2 and the probability of measuring |1> is b^2.

The EPR State

Two qubits are said to be entangled if they are not independent of each other.

Non-entangled: $|00\rangle = |0\rangle \times |0\rangle$



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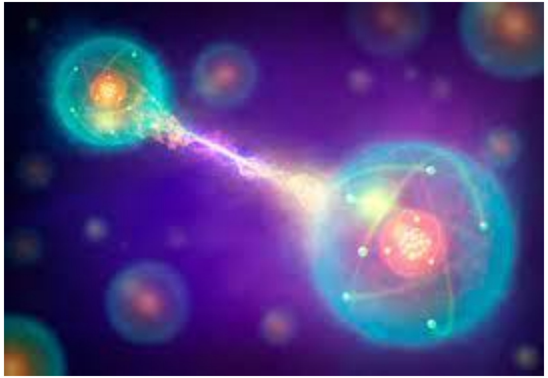

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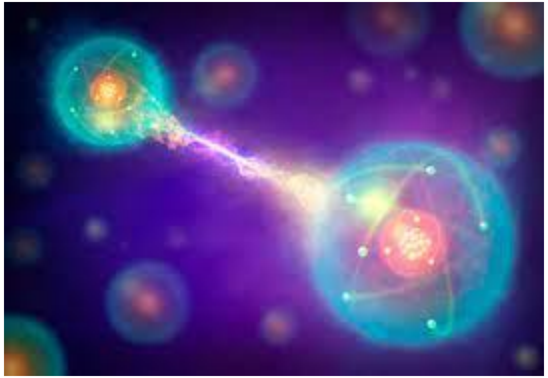
$$|00\rangle + |01\rangle = |0\rangle \times (|0\rangle + |1\rangle)$$



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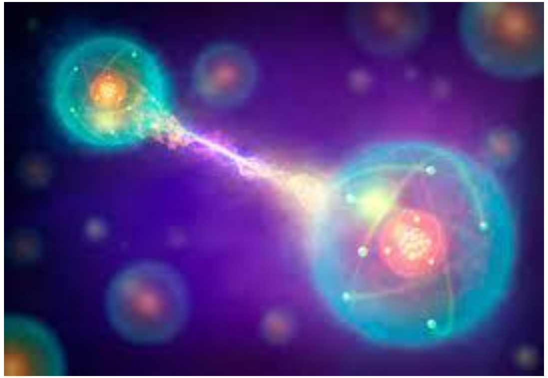
|00> + |01> = |0> × (|0> + |1>)

Entangled: |00> + |11>



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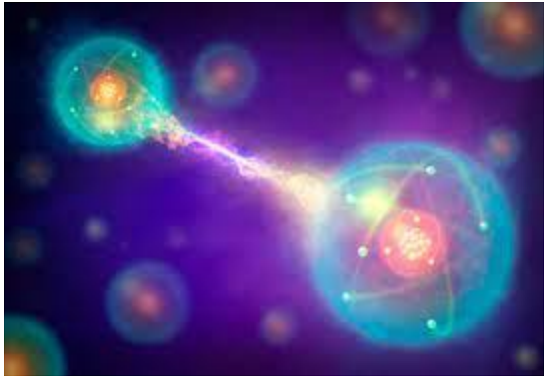

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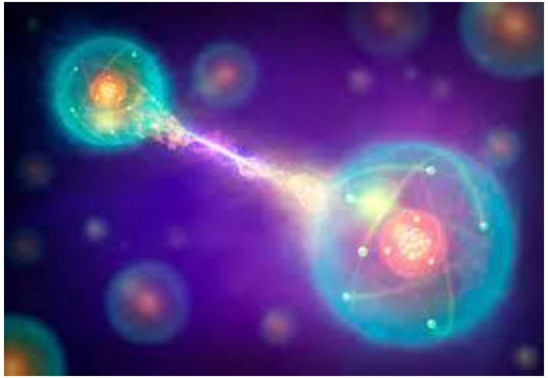

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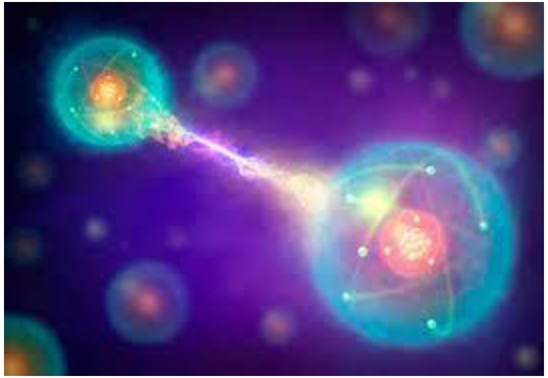

<u>Note</u>: We have not normalised the qubits.

Can Quantum Experiments be simulated using randomness?

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1. Alice and Bob create EPR pair: |00> + |11>
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Can Quantum Experiments be simulated using randomness?

1. Alice and Bob create EPR pair: $|00\rangle + |11\rangle$
2. Alice takes the first qubit with her and leaves Bob with the second qubit.
3. Alice measures her qubit.
4. Bob measures his qubit.

Can Quantum Experiments be simulated using randomness?

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1. a = Random(0,1)
2. b = a
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3. Alice takes A without looking at it and leaves B with Bob.
4. Alice looks at a.
5. Bob looks at b.

Alice and Bob play another game




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Alice and Bob win with probability 75% if the both say they will keep the cash.

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Bell’s Inequality

Alice and Bob can not come up with a better strategy even if they had access to random bits.

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Bell's Inequality

Alice and Bob can not come up with a better strategy even if they had access to random bits.

If Alice and Bob have access to an EPR pair, then there is a strategy that allows them to win with probability at least 80%

Playing around with the EPR Pair

Toggling from a Distance

```
Toggle(|0>) = |1>.
Toggle(|1>) = |0>.
```

Toggling from a Distance

Toggle(|0>) = |1>.
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Suppose Alice and Bob have an EPR pair: |00> + |11>

Toggling from a Distance

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Suppose Alice and Bob have an EPR pair: |00> + |11>

Alice takes the first qubit with her and leaves Bob with the second qubit.

Toggling from a Distance

Toggle(|0>) = |1>.
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Suppose Alice and Bob have an EPR pair: |00> + |11>

Alice takes the first qubit with her and leaves Bob with the second qubit.

State if Alice toggles her qubit: |10> + |01>

Toggling from a Distance

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Toggle(|0>) = |1>.
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State if Bob toggles his qubit: |01> + |10>

Toggling from a Distance

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Suppose Alice and Bob have an EPR pair: |00> + |11>

Alice takes the first qubit with her and leaves Bob with the second qubit.

State if Alice toggles her qubit: |10> + |01>

State if Bob toggles his qubit: |01> + |10>

It is as though Alice can toggle Bob's qubit by toggling her own qubit since they are equivalent operations.

Rotations

$\mathrm{Rot}_\theta(|0\rangle) = \cos(\theta)|0\rangle + \sin(\theta)|1\rangle$

$\mathrm{Rot}_\theta(|1\rangle) = -\sin(\theta)|0\rangle + \cos(\theta)|1\rangle$

Rotations

Suppose Alice and Bob have an EPR pair: |00> + |11>

Alice takes the first qubit with her and leaves Bob with the second qubit.

$$\mathrm{Rot}_\theta(|0\rangle) = \cos(\theta)|0\rangle + \sin(\theta)|1\rangle$$

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Rotations

Suppose Alice and Bob have an EPR pair: |00> + |11>

Alice takes the first qubit with her and leaves Bob with the second qubit.

$Rot_\theta(|0>) = cos(\theta)|0> + sin(\theta)|1>$

$Rot_\theta(|1>) = -sin(\theta)|0> + cos(\theta)|1>$

State if Alice performs Rot_θ on her qubit: $Rot_\theta(|0>)|0> + Rot_\theta(|1>)|1>$

$cos(\theta)|00> + sin(\theta)|10>$
$-sin(\theta)|01> + cos(\theta)|11>$

Rotations

Suppose Alice and Bob have an EPR pair: $|00\rangle + |11\rangle$

Alice takes the first qubit with her and leaves Bob with the second qubit.

$$\mathrm{Rot}_\theta(|0\rangle) = \cos(\theta)|0\rangle + \sin(\theta)|1\rangle$$

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State if Alice performs Rot_θ on her qubit: $\mathrm{Rot}_\theta(|0\rangle)|0\rangle + \mathrm{Rot}_\theta(|1\rangle)|1\rangle$

$$\cos(\theta)|00\rangle + \sin(\theta)|10\rangle$$
$$-\sin(\theta)|01\rangle + \cos(\theta)|11\rangle$$

State if Bob performs Rot_θ on his qubit: $|0\rangle\mathrm{Rot}_\theta(|0\rangle) + |1\rangle\mathrm{Rot}_\theta(|1\rangle)$

$$\cos(\theta)|00\rangle + \sin(\theta)|01\rangle$$
$$-\sin(\theta)|10\rangle + \cos(\theta)|11\rangle$$

Rotations

Suppose Alice and Bob have an EPR pair: $|00\rangle + |11\rangle$

Alice takes the first qubit with her and leaves Bob with the second qubit.

Note: $\cos(-\theta)=\cos(\theta)$ and $\sin(-\theta)=-\sin(\theta)$

$$\mathrm{Rot}_\theta(|0\rangle) = \cos(\theta)|0\rangle + \sin(\theta)|1\rangle$$

$$\mathrm{Rot}_\theta(|1\rangle) = -\sin(\theta)|0\rangle + \cos(\theta)|1\rangle$$

State if Alice performs Rot_θ on her qubit: $\mathrm{Rot}_\theta(|0\rangle)|0\rangle + \mathrm{Rot}_\theta(|1\rangle)|1\rangle$

$$\cos(\theta)|00\rangle + \sin(\theta)|10\rangle$$
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Rotations

Suppose Alice and Bob have an EPR pair: |00> + |11>

Alice takes the first qubit with her and leaves Bob with the second qubit.

Note: cos(-θ)=cos(θ) and sin(-θ)=-sin(θ)

It is as though Alice can simulate Bob performing Rot_θ on his qubit by performing $Rot_{-\theta}$ on her own qubit.

$Rot_\theta(|0\rangle) = \cos(\theta)|0\rangle + \sin(\theta)|1\rangle$

$Rot_\theta(|1\rangle) = -\sin(\theta)|0\rangle + \cos(\theta)|1\rangle$

State if Alice performs Rot_θ on her qubit: $Rot_\theta(|0\rangle)|0\rangle + Rot_\theta(|1\rangle)|1\rangle$

$\cos(\theta)|00\rangle + \sin(\theta)|10\rangle$
$-\sin(\theta)|01\rangle + \cos(\theta)|11\rangle$

State if Bob performs Rot_θ on his qubit: $|0\rangle Rot_\theta(|0\rangle) + |1\rangle Rot_\theta(|1\rangle)$

$\cos(\theta)|00\rangle + \sin(\theta)|01\rangle$
$-\sin(\theta)|10\rangle + \cos(\theta)|11\rangle$

Law of Quantum Transformations

Law of Quantum Transformations

$2^k \times 2^k$ Unitary Matrices

exactly capture all physically possible transformations

that can be performed on k-length qubits.

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|0> is equivalent to $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

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Toggle(|x>) $\asymp$ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Law of Quantum Transformations

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$|0\rangle$ is equivalent to $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$|1\rangle$ is equivalent to $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\text{Toggle}(|x\rangle) \asymp \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\text{Rot}_\theta(|0\rangle) \asymp \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$

A better Quantum Strategy

Alice and Bob play another game

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep




Alice and Bob win with probability 75% if the both say they will keep the cash.

Bell’s Inequality
Alice and Bob can not come up with a better strategy even if they had access to random bits.

Alice and Bob play another game




Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
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Alice and Bob win with probability 75% if the both say they will keep the cash.

Bell's Inequality

Alice and Bob can not come up with a better strategy even if they had access to random bits.

If Alice and Bob have access to an EPR pair, then there is a strategy that allows them to win with probability at least 80%

Alice and Bob play The CHSH Game

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep




Quantum Strategy

Alice and Bob play The CHSH Game

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by ($-\pi/8$).

ALICE AND BOB PLAY THE CHSH GAME

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep




Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

Alice and Bob play The CHSH Game

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
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Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep




Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by ($-\pi/8$).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by ($\pi/8$).

Alice and Bob play The CHSH Game

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
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Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep




<u>Quantum Strategy</u>

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

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Alice and Bob play The CHSH Game

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
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Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Claim: With this strategy, Alice and will win with probability at least 80%




Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by $(-\pi/8)$.

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by $(\pi/8)$.

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Alice and Bob play The CHSH Game

Case 1: Coin Toss Result for Alice and Bob is Tail

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 1: Coin Toss Result for Alice and Bob is Tail

In this case, both Alice and Bob directly measure their qubits without performing any transformations.

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (-π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 1: Coin Toss Result for Alice and Bob is Tail

In this case, both Alice and Bob directly measure their qubits without performing any transformations.

Since their qubits are entangled, this would mean their answers would always match.

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 1: Coin Toss Result for Alice and Bob is Tail

In this case, both Alice and Bob directly measure their qubits without performing any transformations.

Since their qubits are entangled, this would mean their answers would always match.

So in this case, Alice and Bob always win.

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 2: Coin Toss Result for Alice is Tail and for Bob is Head

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 2: Coin Toss Result for Alice is Tail and for Bob is Head

In this case, Alice measures her qubit without performing any transformations.

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 2: Coin Toss Result for Alice is Tail and for Bob is Head

In this case, Alice measures her qubit without performing any transformations.

But Bob measures his qubit after rotation by (π/8).

<u>**Quantum Strategy**</u>

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 2: Coin Toss Result for Alice is Tail and for Bob is Head

In this case, Alice measures her qubit without performing any transformations.

But Bob measures his qubit after rotation by $(\pi/8)$. So the (unnormalised) state before measurement is

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by $(-\pi/8)$.

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by $(\pi/8)$.

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 2: Coin Toss Result for Alice is Tail and for Bob is Head

In this case, Alice measures her qubit without performing any transformations.

But Bob measures his qubit after rotation by (π/8). So the (unnormalised) state before measurement is

$$\cos(\pi/8)|00\rangle + \sin(\pi/8)|01\rangle - \sin(\pi/8)|10\rangle + \cos(\pi/8)|11\rangle$$

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 2: Coin Toss Result for Alice is Tail and for Bob is Head

The (unnormalised) state before measurement is

$$\cos(\pi/8)|00\rangle + \sin(\pi/8)|01\rangle$$
$$-\sin(\pi/8)|10\rangle + \cos(\pi/8)|11\rangle$$

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 2: Coin Toss Result for Alice is Tail and for Bob is Head

The (unnormalised) state before measurement is

$$\cos(\pi/8)|00\rangle + \sin(\pi/8)|01\rangle$$
$$-\sin(\pi/8)|10\rangle + \cos(\pi/8)|11\rangle$$

So the probability that both answer consistently is

<u>Quantum Strategy</u>

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 2: Coin Toss Result for Alice is Tail and for Bob is Head

The (unnormalised) state before measurement is

$$\cos(\pi/8)|00\rangle + \sin(\pi/8)|01\rangle$$
$$-\sin(\pi/8)|10\rangle + \cos(\pi/8)|11\rangle$$

So the probability that both answer consistently is

$$\cos^2(\pi/8) >= .85$$

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 3: Coin Toss Result for Alice is Head and for Alice is Tail

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 3: Coin Toss Result for Alice is Head and for Alice is Tail

Exact same calculation as in Case 2 shows that the probability that both answer consistently is

$$\cos^2(\pi/8) >= .85$$

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Alice and Bob play The CHSH Game

Case 4: Coin Toss Result for Alice and Bob is Head

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep

Alice and Bob play The CHSH Game

Case 4: Coin Toss Result for Alice and Bob is Head

In this case, both Alice and Bob measure their qubits after rotating them by (−π/8), (π/8) respectively.

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep

Alice and Bob play The CHSH Game

Case 4: Coin Toss Result for Alice and Bob is Head

In this case, both Alice and Bob measure their qubits after rotating them by (-π/8), (π/8) respectively.

One can check that after these transformations, the probability of observing each of the basic states |00>, |01>, |10> and |11> is equal.

Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (-π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep

Alice and Bob play The CHSH Game

Case 4: Coin Toss Result for Alice and Bob is Head

In this case, both Alice and Bob measure their qubits after rotating them by (-π/8), (π/8) respectively.

One can check that after these transformations, the probability of observing each of the basic states |00>, |01>, |10> and |11> is equal.

Thus, the probability of Alice and Bob answering differently is 0.5

<u>Quantum Strategy</u>

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (-π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by (π/8).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
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Alice and Bob play The CHSH Game

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep




Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by ($-\pi/8$).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by ($\pi/8$).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

ALICE AND BOB PLAY THE CHSH GAME

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
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Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Win Probabilty is at least




Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by (−π/8).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

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Alice and Bob play The CHSH Game

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
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Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Win Probabilty is at least
0.25×1




Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by $(-\pi/8)$.

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by $(\pi/8)$.

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Alice and Bob play The CHSH Game

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Win Probabilty is at least
$0.25 \times 1 + 0.5 \times 0.85$




Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by $(-\pi/8)$.

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by $(\pi/8)$.

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Alice and Bob play The CHSH Game

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Win Probabilty is at least
$0.25 \times 1 + 0.5 \times 0.85 + 0.25 \times 0.5$




Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by $(-\pi/8)$.

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by $(\pi/8)$.

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

Alice and Bob play The CHSH Game

Coin Toss Result (Alice)	Coin Toss Result (Bob)	Winning Situations
Head	Head	Alice -> Keep, Bob -> Not keep Alice -> Not keep, Bob -> Keep
Head	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Head	Alice, Bob -> Not keep Alice, Bob -> Keep
Tail	Tail	Alice, Bob -> Not keep Alice, Bob -> Keep

Win Probabilty is at least
$0.25 \times 1 + 0.5 \times 0.85 + 0.25 \times 0.5 = 0.8$




Quantum Strategy

If toss result is tails, Alice does nothing. Otherwise she rotates her qubit by ($-\pi/8$).

She then measures her qubit. If it is 1, she responds “Keep”. Else she responds “Not Keep”.

If toss result is tails, Bob does nothing. Otherwise he rotates his qubit by ($\pi/8$).

He then measures his qubit. If it is 1, he responds “Keep”. Else he responds “Not Keep”.

The CHSH Game Proves that there are experiments in which quantum Algorithms cannot be simulated by classical randomised algorithms.

Theoretical Computer Science

- A Problem that needs to be solved.

- A Problem that needs to be solved.
- A Mathematical Model that describes the abilities/restrictions of the solver.

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- A Mathematical Model that describes the abilities/restrictions of the solver.

Study the amount of resources needed by the model to solve the problem.

I am more interested in solving the problem

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- Finding factors of a given number.
- Finding the shortest path between two vertices in a graph.
- Finding the next best move in a chess game.
- Finding out if the given image is that of a dog or a cat.
- Predicting the next word in a sentence.
- .. ..

I am more interested in solving the problem

I am more interested in understanding the model

- Finding factors of a given number.
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- Finding the next best move in a chess game.
- Finding out if the given image is that of a dog or a cat.
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- .. ..

I am more interested in understanding the model

- Communication models.
  - two-party/multi-party
  - compromised channels/uncompromised channels
  - broadcast/point-to-point
- Computers (a.k.a Turing machines)
- Quantum Computers.
- Circuits.
- Proof Systems.
- Prover-Verifier Games.
- .. ..

Complexity Theory

Knowing the limitations

Resources: Time, Space, Randomness

Knowing the limitations

Resources: Time, Space, Randomness, Number of classical gates

Knowing the limitations

Resources: Time, Space, Randomness, Number of classical gates, Number of Quantum gates

Knowing the limitations

Resources: Time, Space, Randomness, Number of classical gates, Number of Quantum gates, Number of lines in the proof

Knowing the limitations

Resources: Time, Space, Randomness, Number of classical gates, Number of Quantum gates, Number of lines in the proof, Number of queries asked

Knowing the limitations

Resources: Time, Space, Randomness, Number of classical gates, Number of Quantum gates, Number of lines in the proof, Number of queries asked, Number of bits exchanged.

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Amount of resources needed by Model B to solve Problem A is at least <something large>.

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Cryptography

SCoS @ NISER

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Secure Multiparty Computation

Secure multiparty computation (MPC) is a cryptographic protocol that **allows multiple parties to jointly compute a function** of their inputs while revealing as little as possible about those inputs.

Some of the applications of MPC are **online auctions, voting** etc.

Machine Learning

Machine Learning (ML) is a type of artificial intelligence (AI) that allows software applications to become more accurate in **predicting outcomes without being explicitly programmed to do so**.

Application of Machine learning includes **healthcare, natural language processing, recommendation systems.**

Data Clustering

Data clustering is the process of **grouping data points together based on their similarities**. Clustering is an unsupervised learning technique.

Application of data clustering includes **market segmentation, image segmentation, anomaly detection** and many more.

Algorithm Design

Algorithm Design refers to developing efficient algorithms to **solve computational problems and analysing their complexity**.

The aim is to devise algorithms that optimize time and space complexities, addressing challenges in various domains like **optimization, cryptography, artificial intelligence**, and more.

Complexity Theory

Complexity Theory is the study of different **computational models**.

Research in this area focuses on understanding the **power and limitations** of objects that model **real-world machines** with varied restrictions. Knowing the limitations of a model helps us devise **secure protocols** against them.

THANK YOU !

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