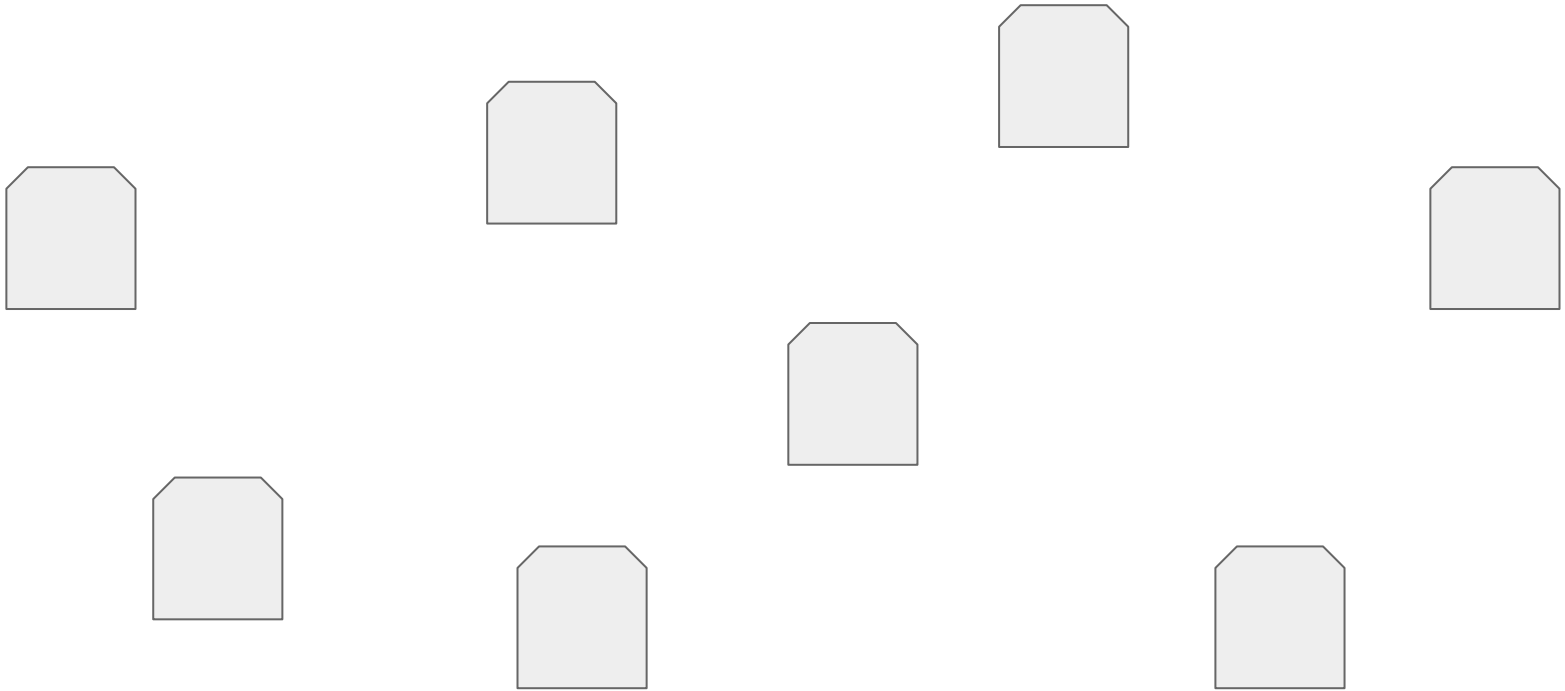
How hard is it to solve this?

Prerona Chatterjee

NISER Bhubaneswar

Connecting villages



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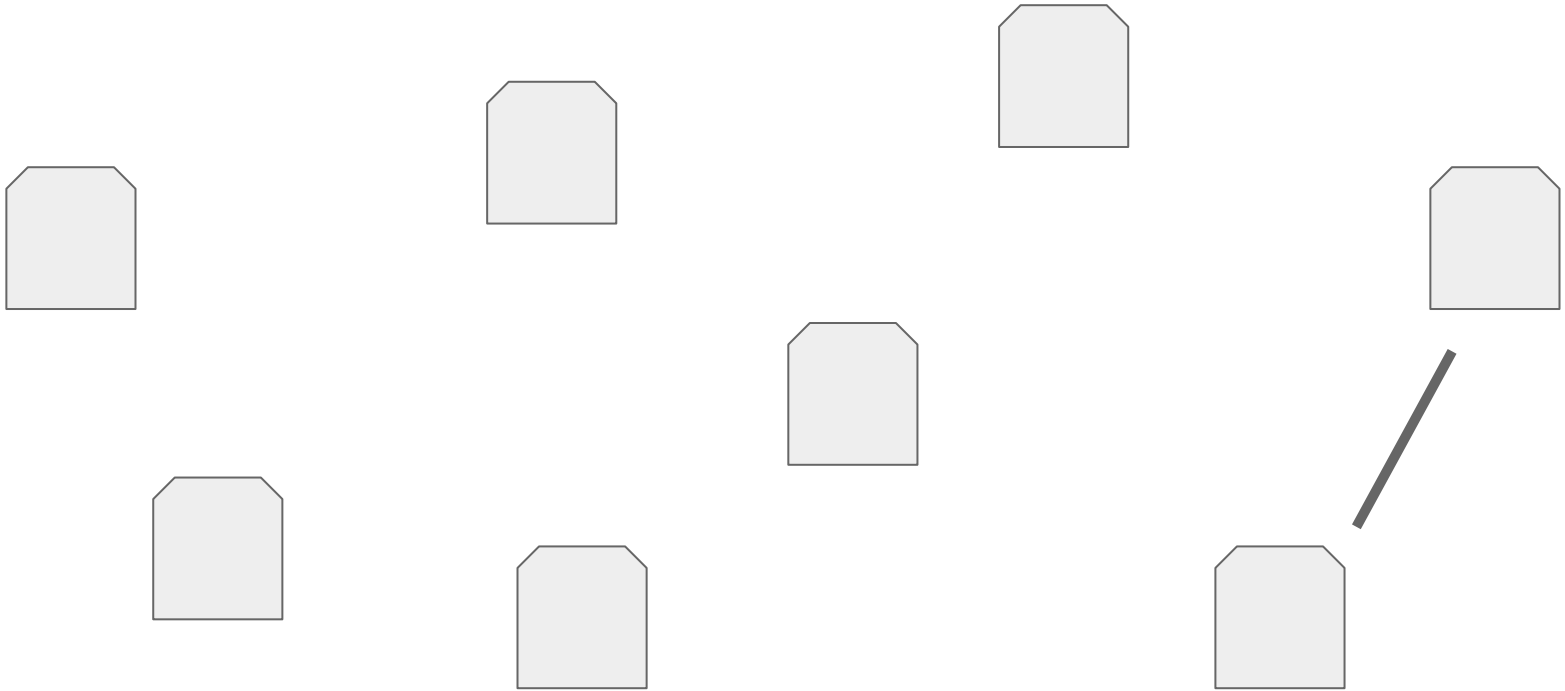

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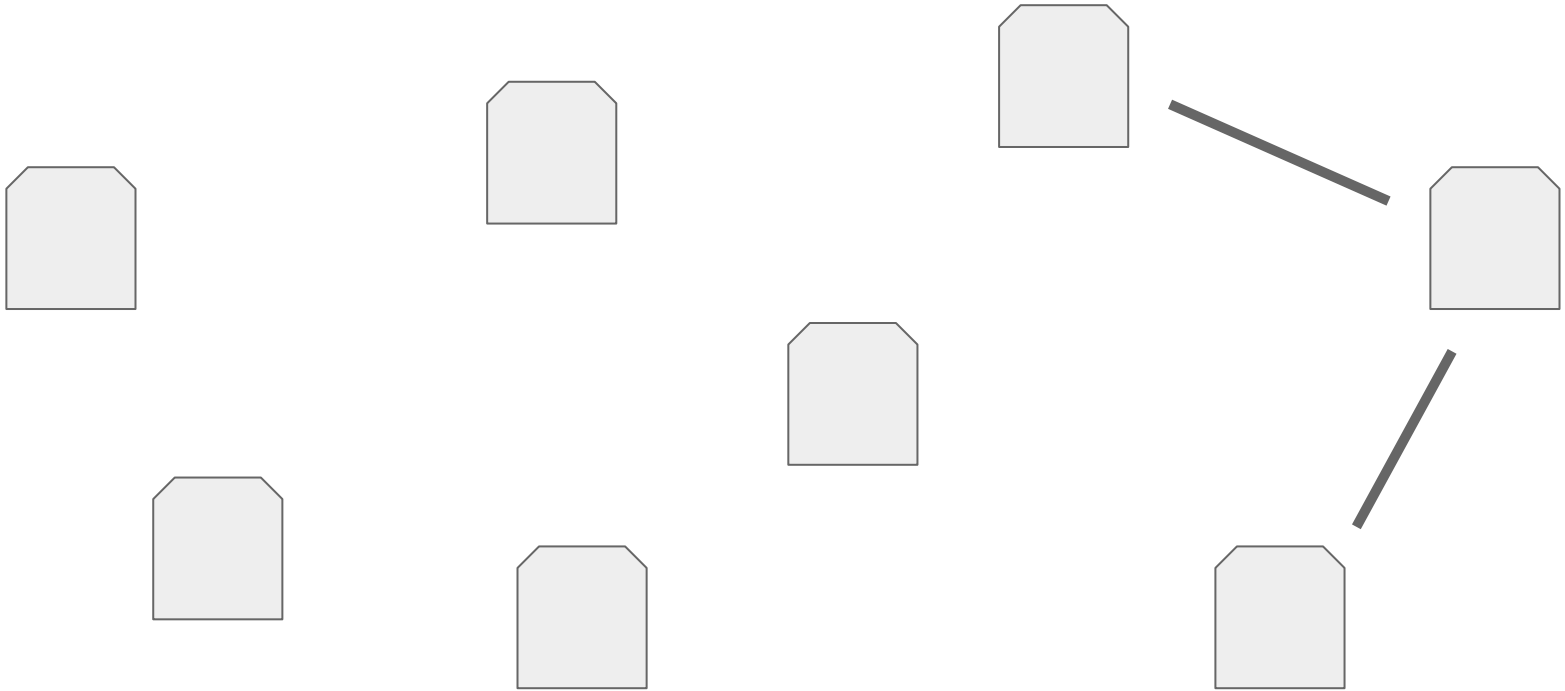

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- If no, why not? How much budget would be enough?

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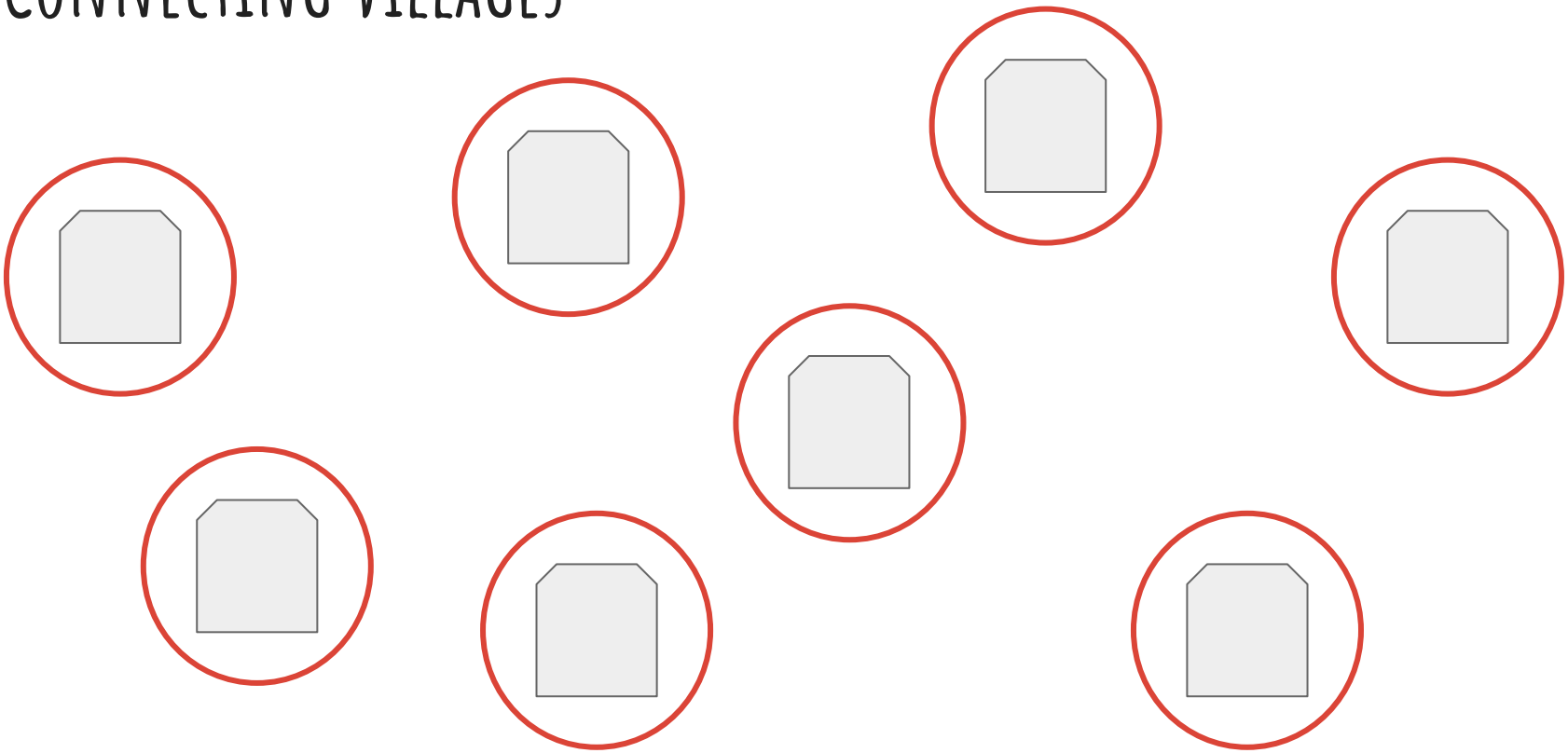
₹70,000 is enough. But is it necessary?

Connecting villages



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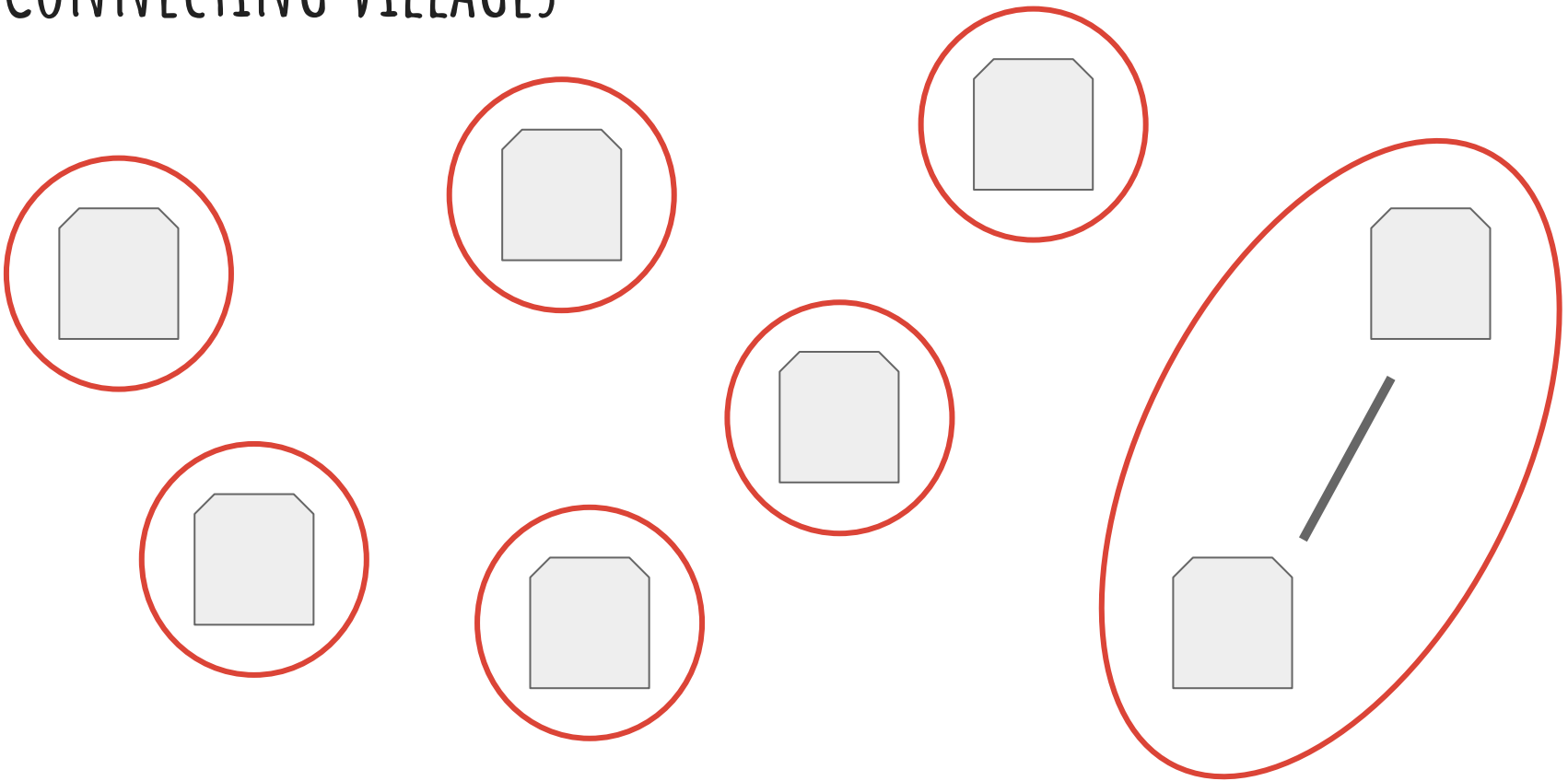

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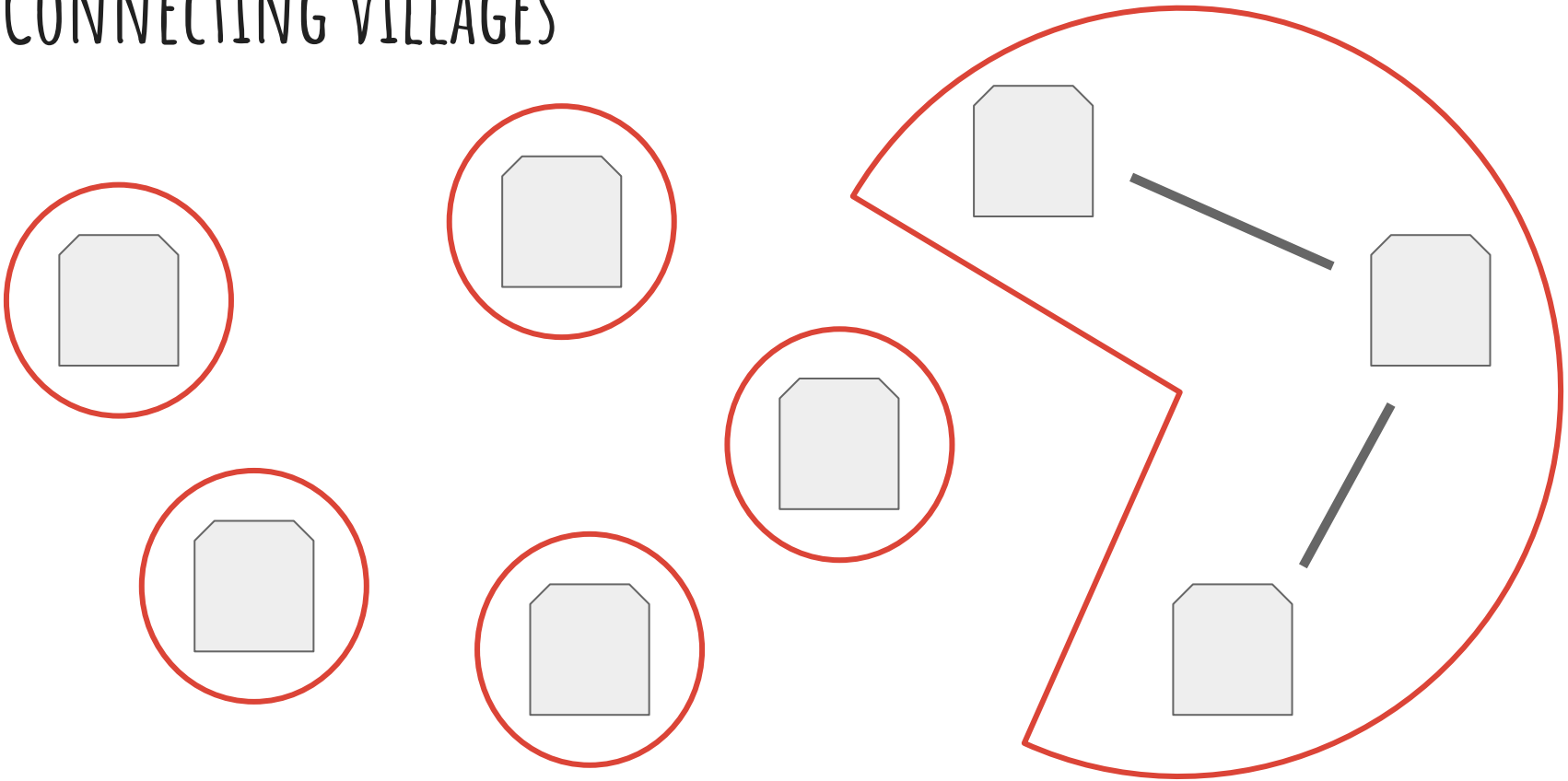

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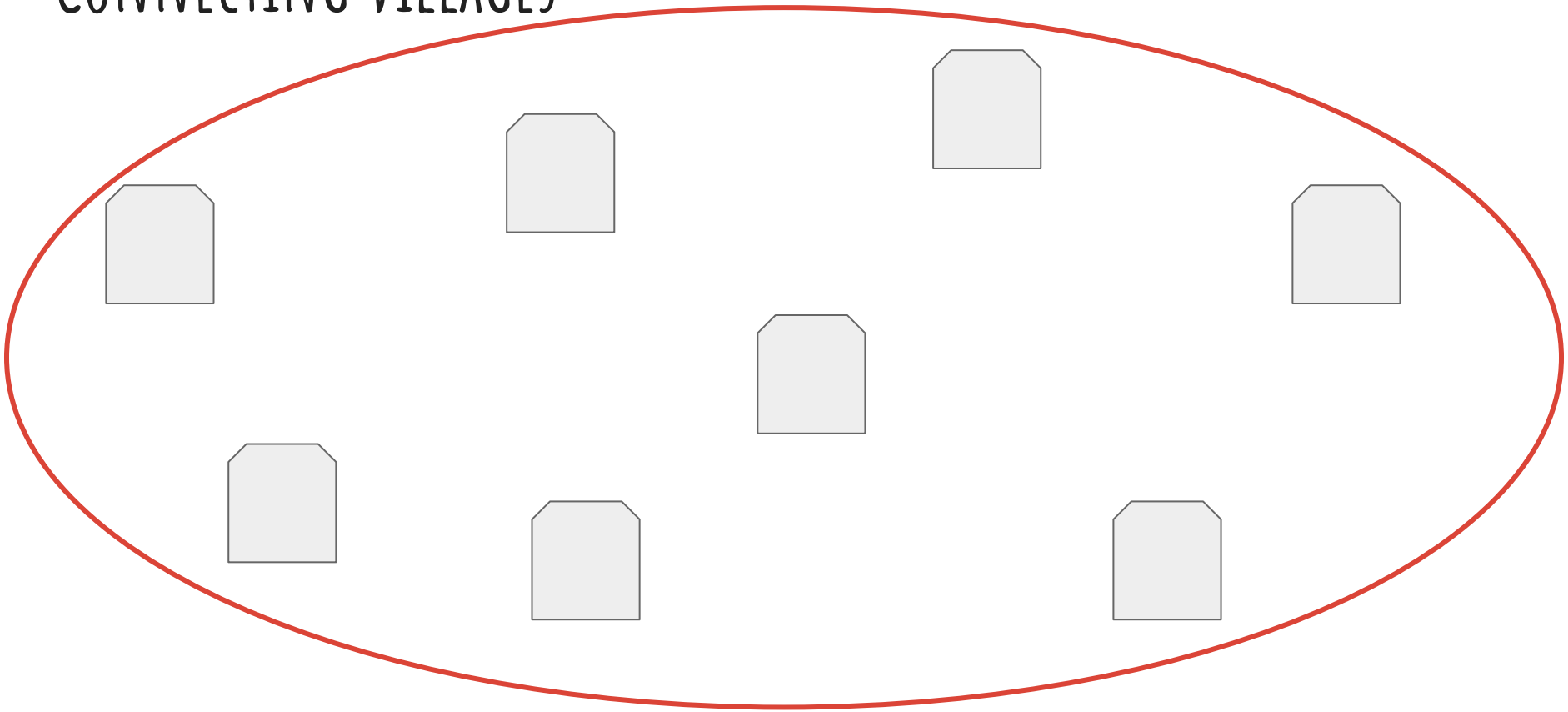

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₹70,000 is indeed necessary.

How is proving that something is hard helpful?

Knowing the limitations

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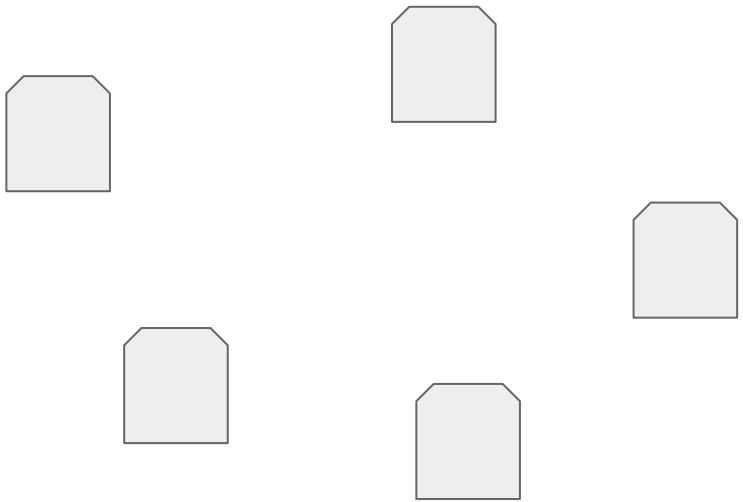
- Is it necessary to find the exact solution?

The travelling salesperson



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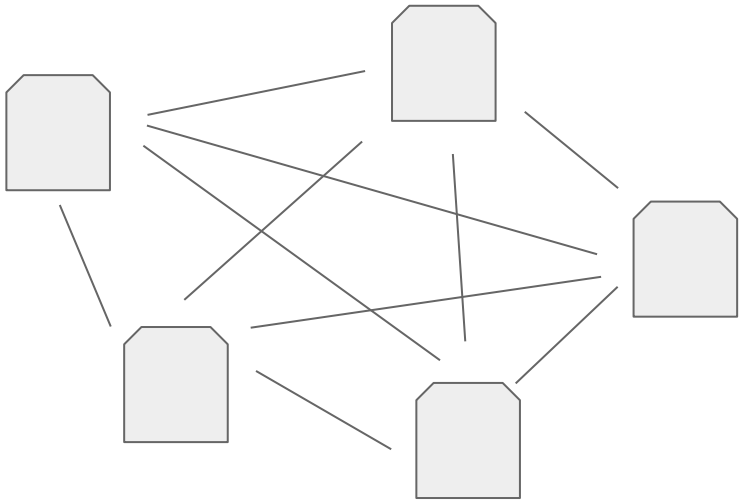

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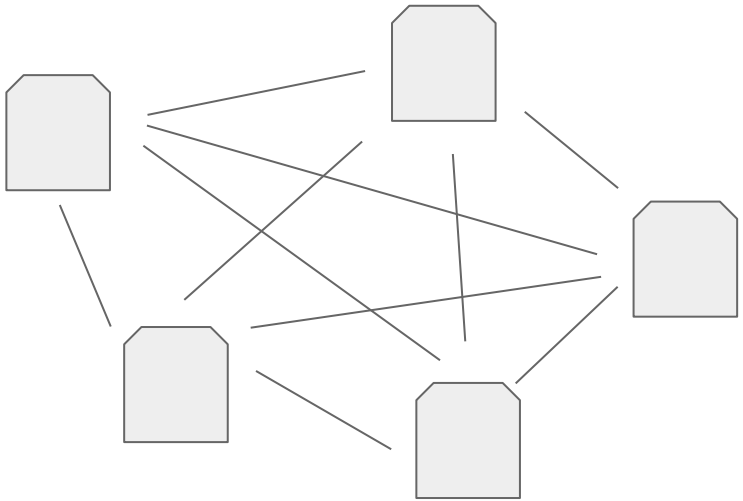

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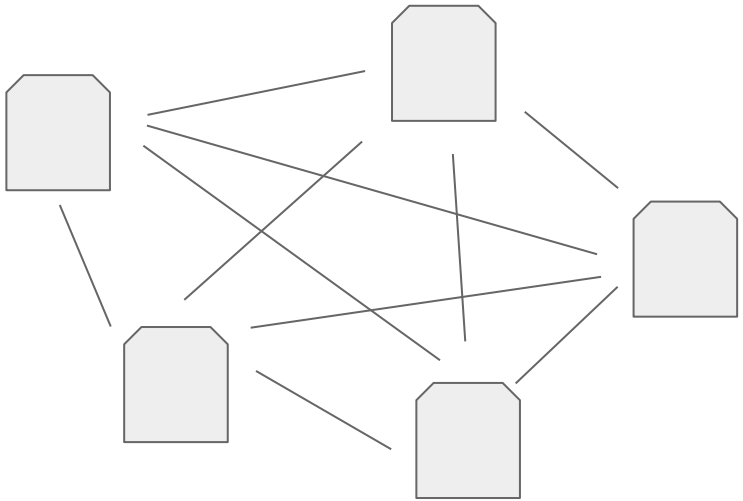

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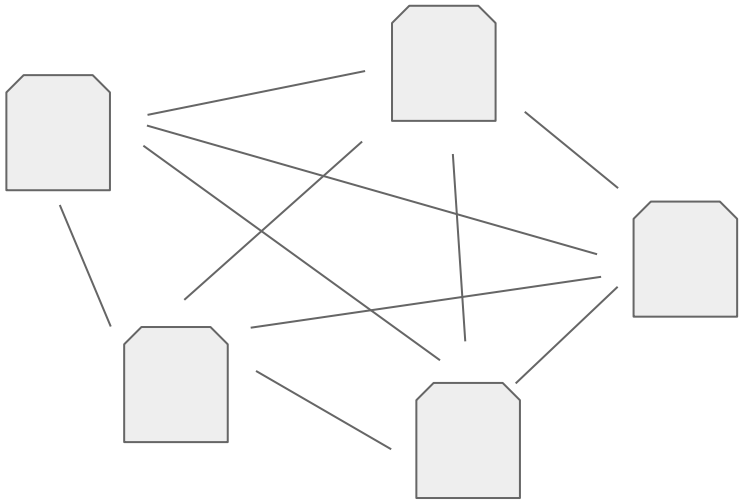

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- Find the route which minimises the distance.

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Approximation algorithms

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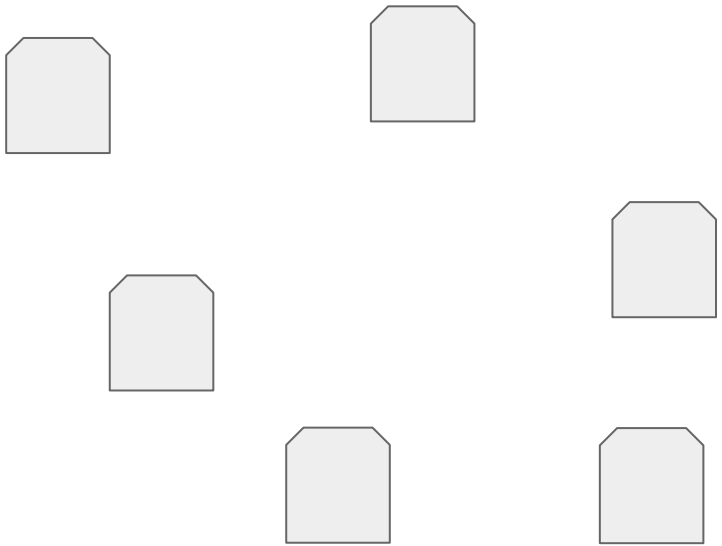
k-Vertex Cover



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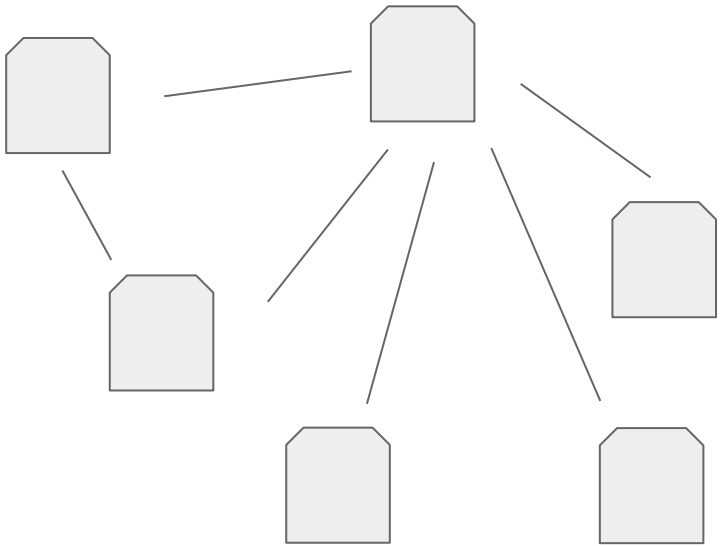

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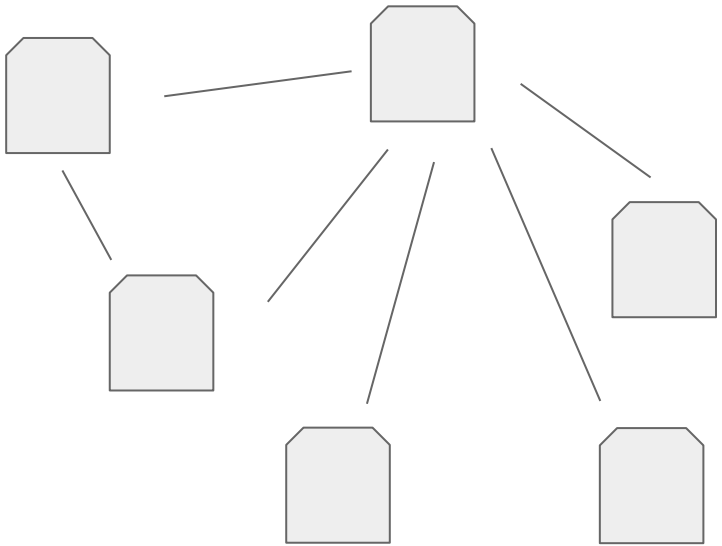

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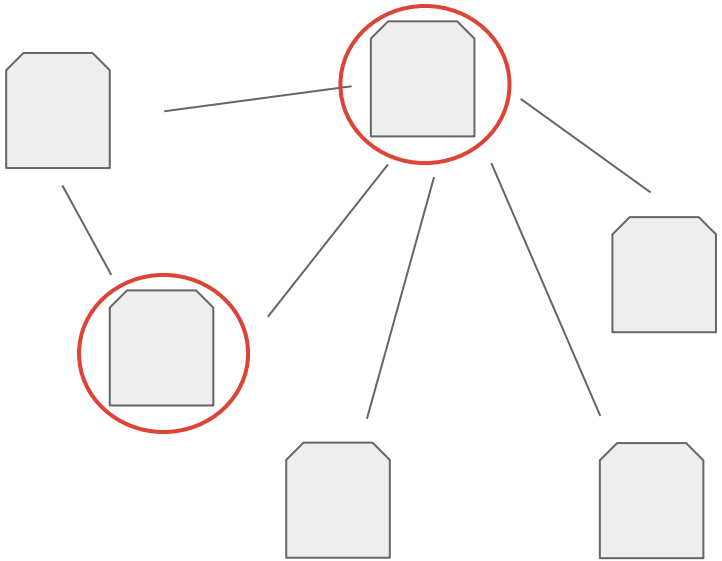

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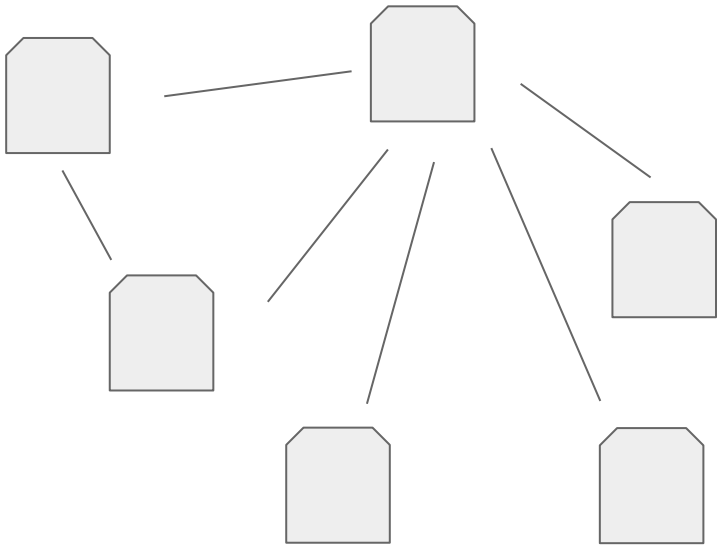

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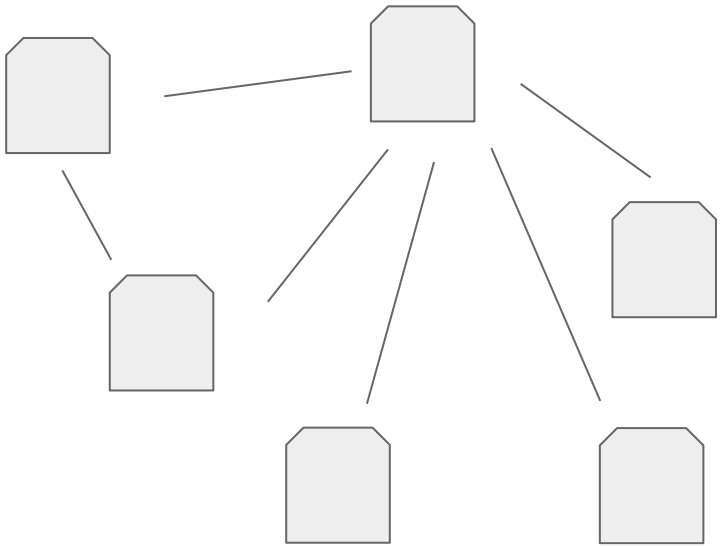

- All the villages are connected to each other, but not necessarily by a direct road.
- Want to place police stations in some villages so that each road is guarded by some police station.
- For a given k, find out if it is possible to do this with only k police stations. Output the villages they should be constructed in.

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Parameterised algorithms

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Secretly adding numbers

- Create groups of 5.
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As a group, you want to find the sum of the numbers selected by your members, WITHOUT ANY OF YOU FINDING OUT THE NUMBERS SELECTED BY THE OTHERS.

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Sending secret messages

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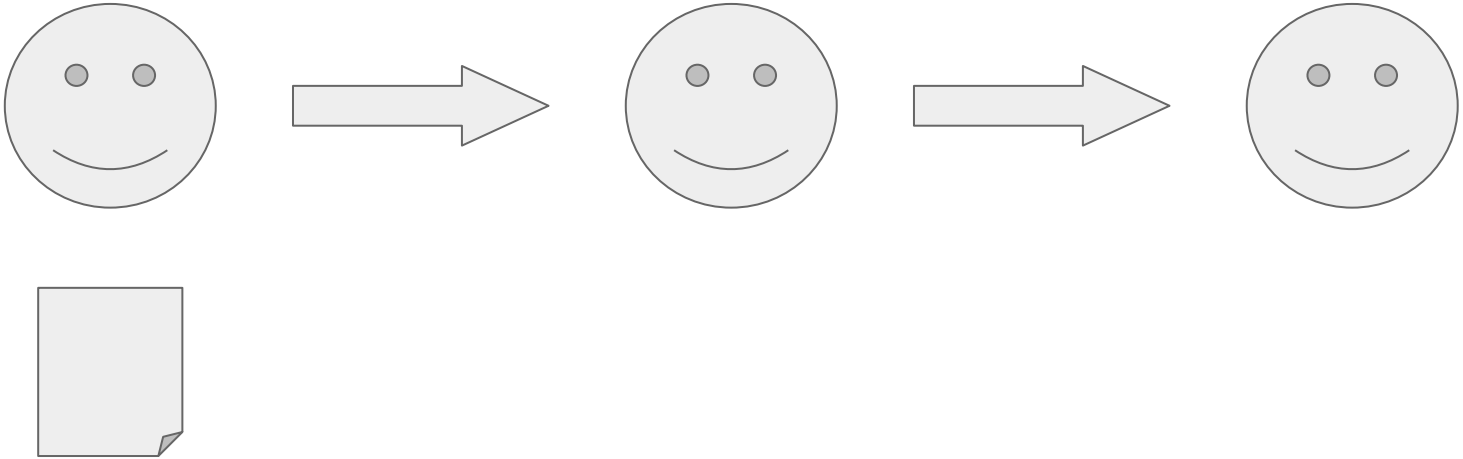

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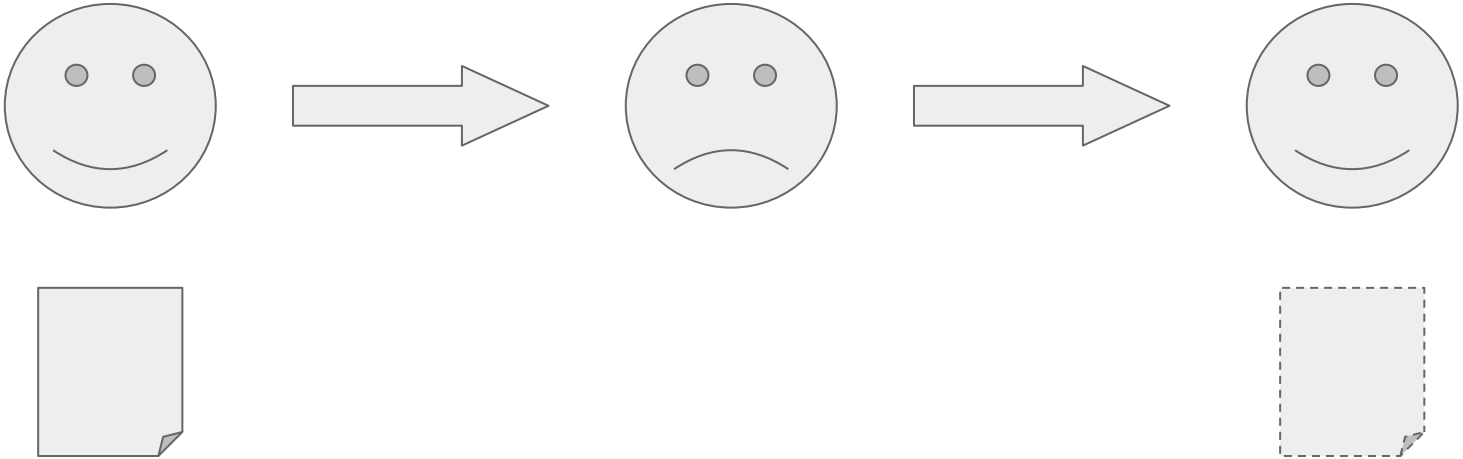

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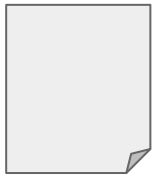

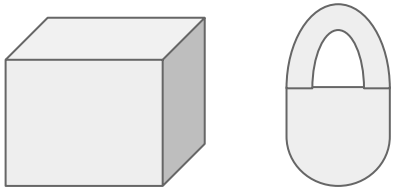

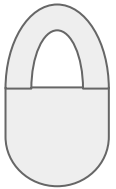

Sending secret messages

- A puts the note in the box and locks it. Gives it to B.

Sending secret messages

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This protocol is secure assuming B does not have any equipment to break the lock.

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Cryptography

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G : Func. s.t. G(x,F(y)) = G(y,F(x))

Theoretical Computer Science

- A Problem that needs to be solved.

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- A Mathematical Model that describes the abilities/restrictions of the solver.

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Study the amount of resources needed by the model to solve the problem.

I am more interested in solving the problem

I AM MORE INTERESTED IN SOLVING THE PROBLEM

- Finding factors of a given number.
- Finding the shortest path between two vertices in a graph.
- Finding the next best move in a chess game.
- Finding out if the given image is that of a dog or a cat.
- Predicting the next word in a sentence.
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- Communication models.
  - two-party/multi-party
  - compromised channels/uncompromised channels
  - broadcast/point-to-point
- Computers (a.k.a Turing machines)
- Quantum Computers.
- Circuits.
- Proof Systems.
- Prover-Verifier Games.
- .. ..

Algebraic Complexity Theory

Understanding the complexity of algebraic problems

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- Finding factors of a given number/polynomial.
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- Multiplying two numbers/polynomials/matrices.
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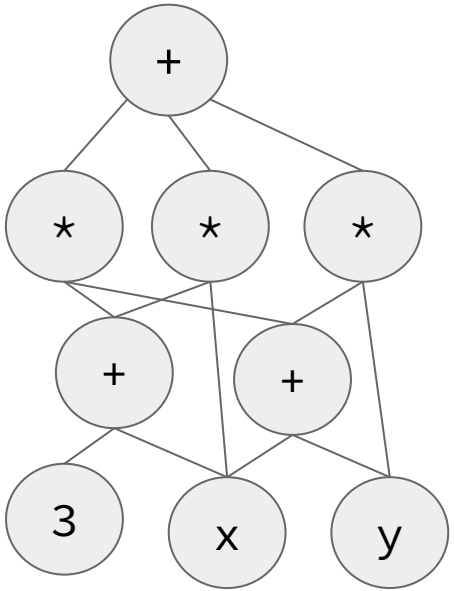

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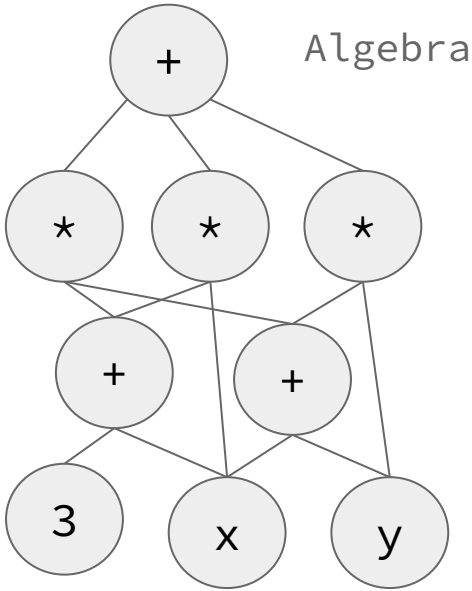

Algebraic Circuits

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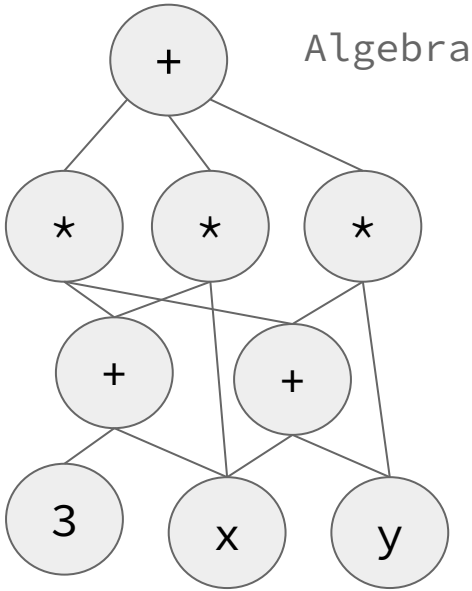
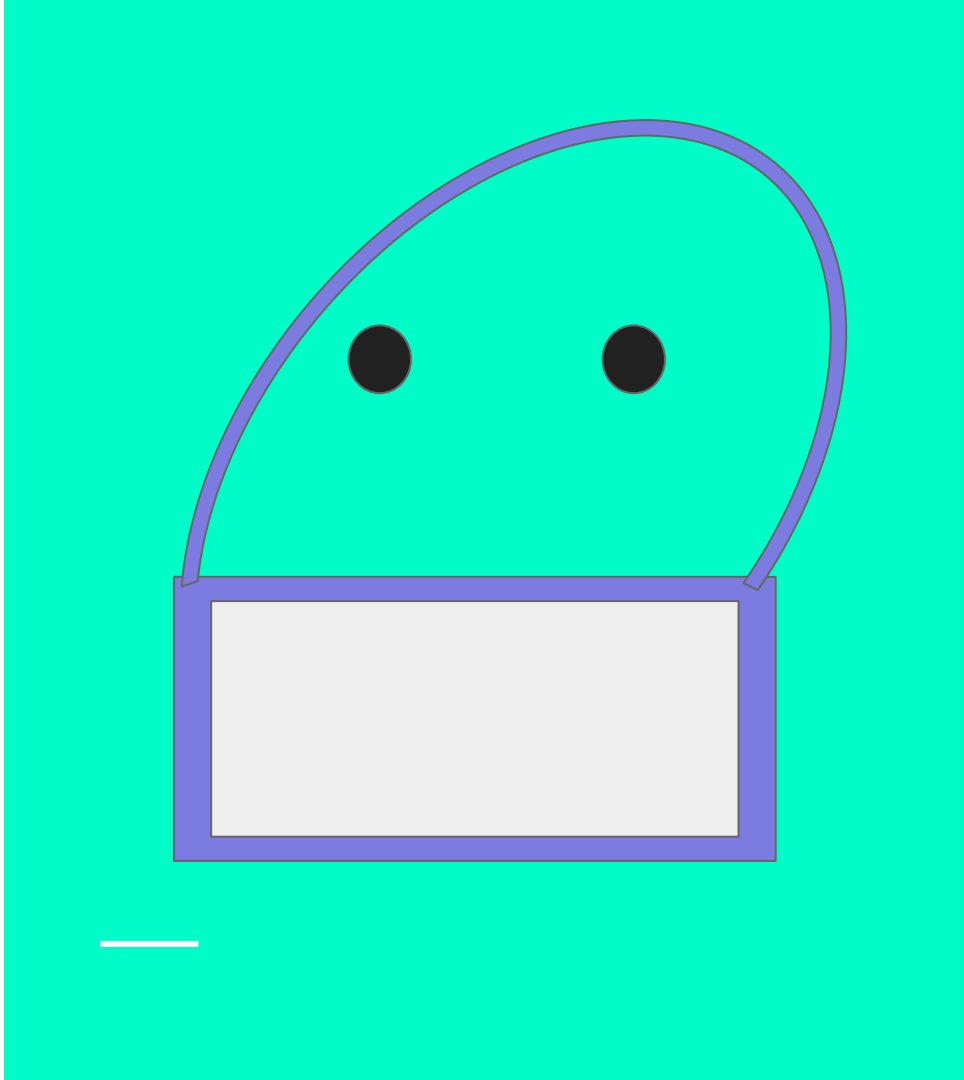


Algebraic Circuits

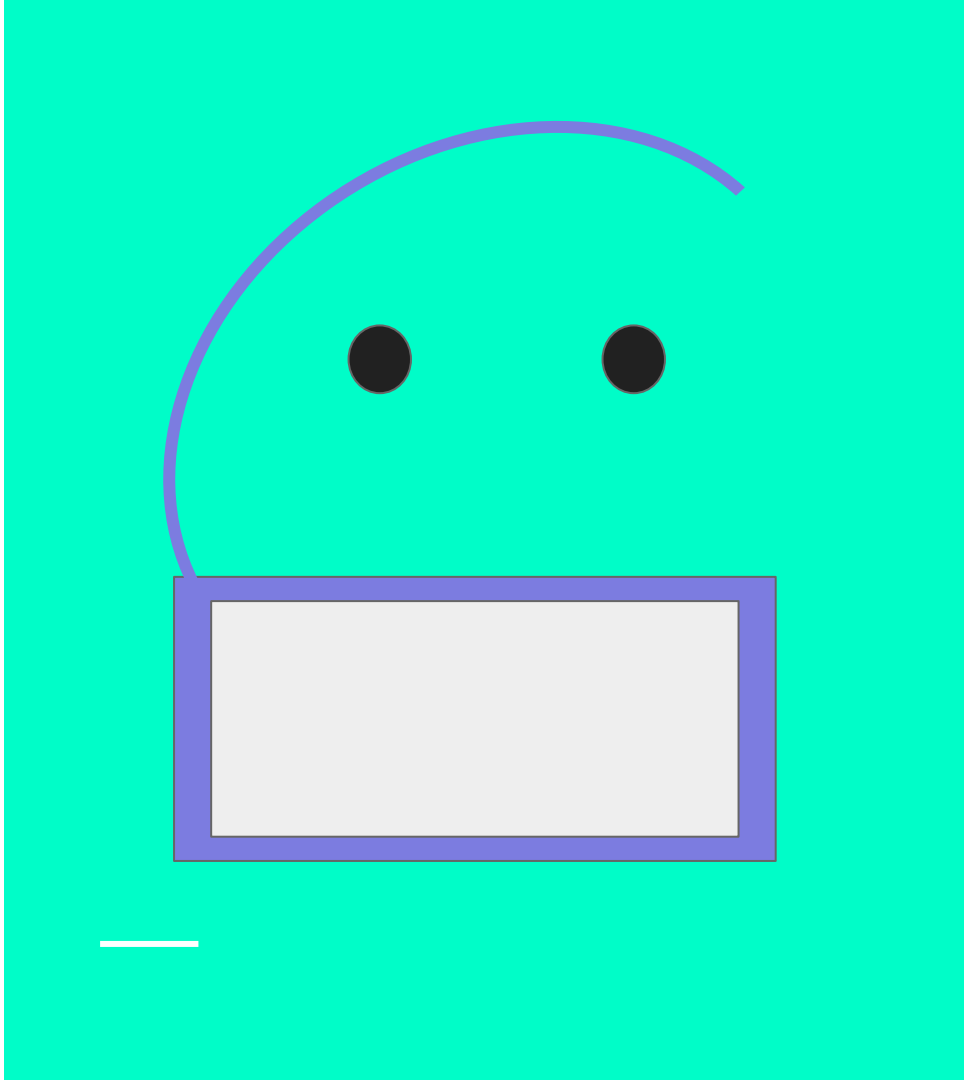
- Algebraic Formulas
- Algebraic Branching Programs
- BSS Model

The Picture Hanging Puzzle



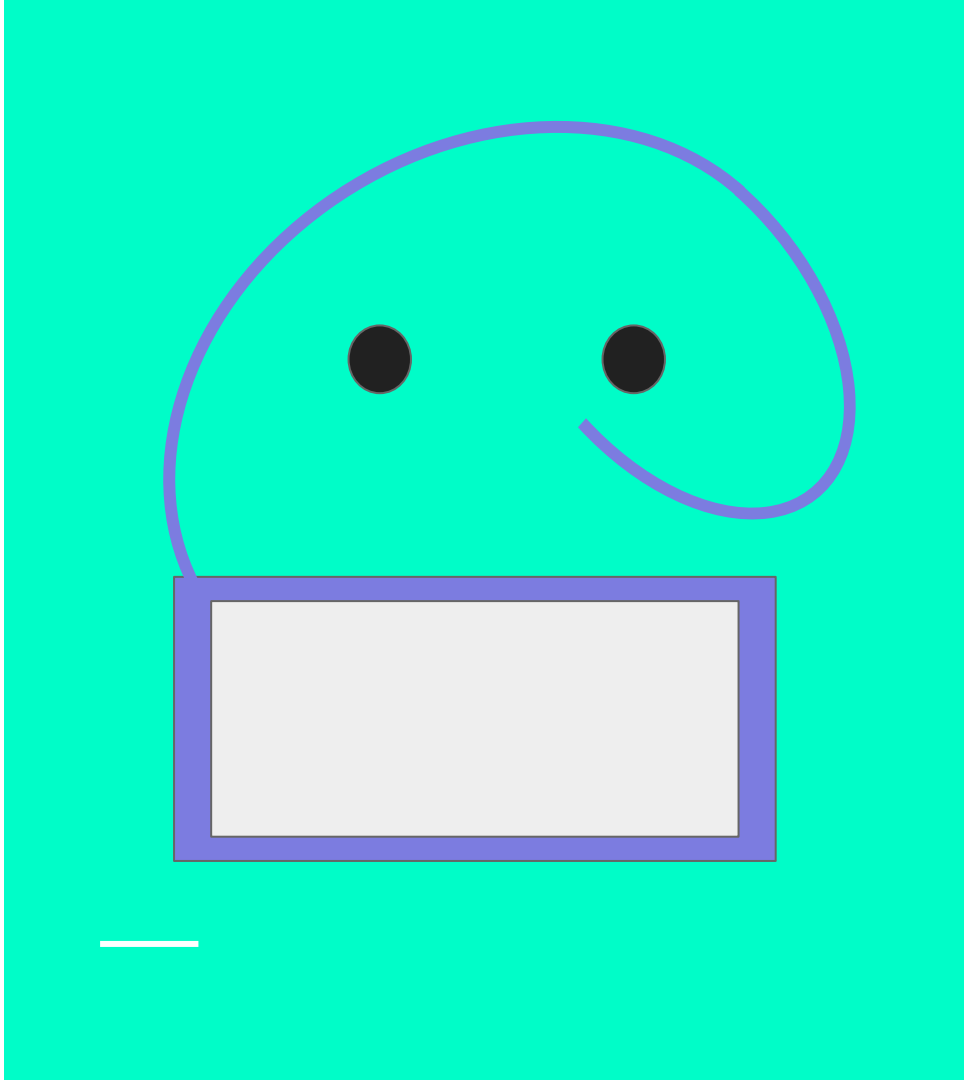
The Picture Hanging Puzzle

Solution: **A**



The Picture Hanging Puzzle

Solution: **AB**



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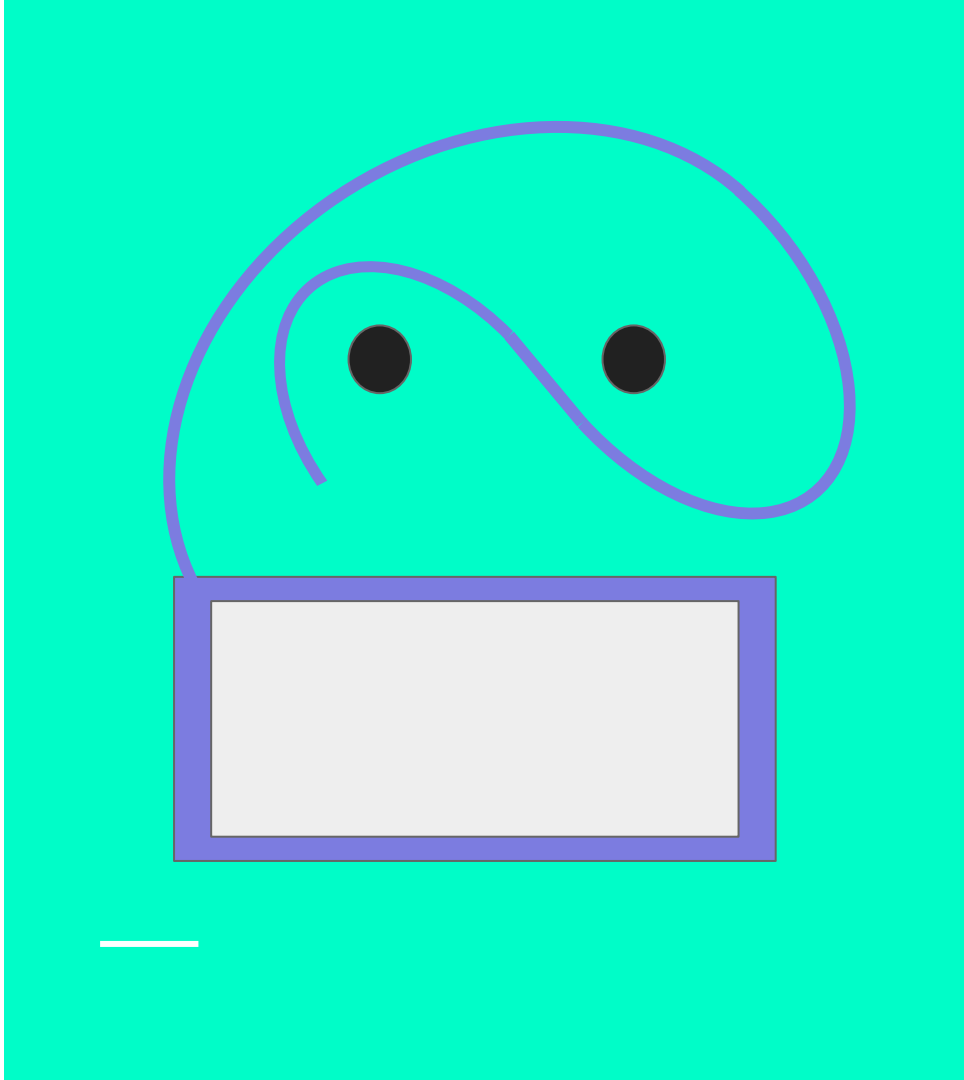
Solution: $\mathbf{ABA^{-1}}$



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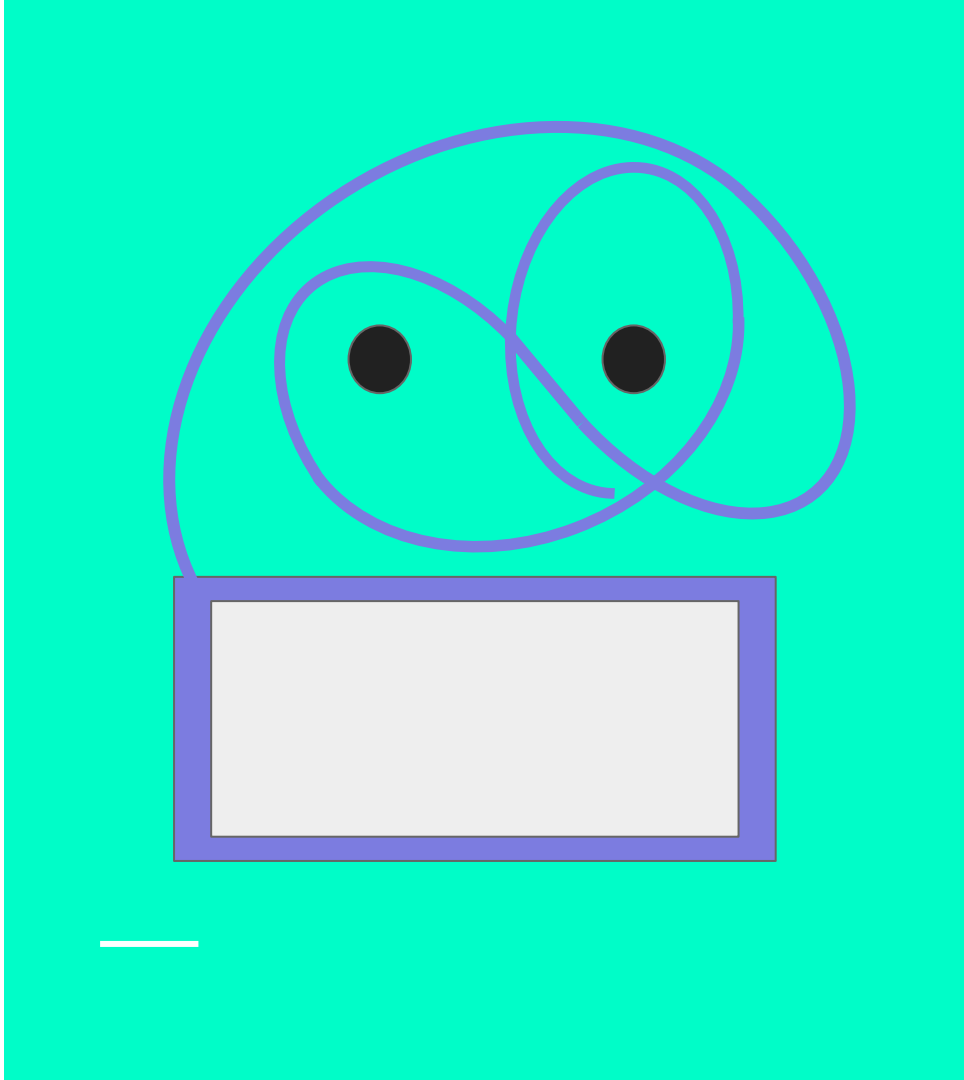
Solution: $\mathbf{ABA^{-1}B^{-1}}$



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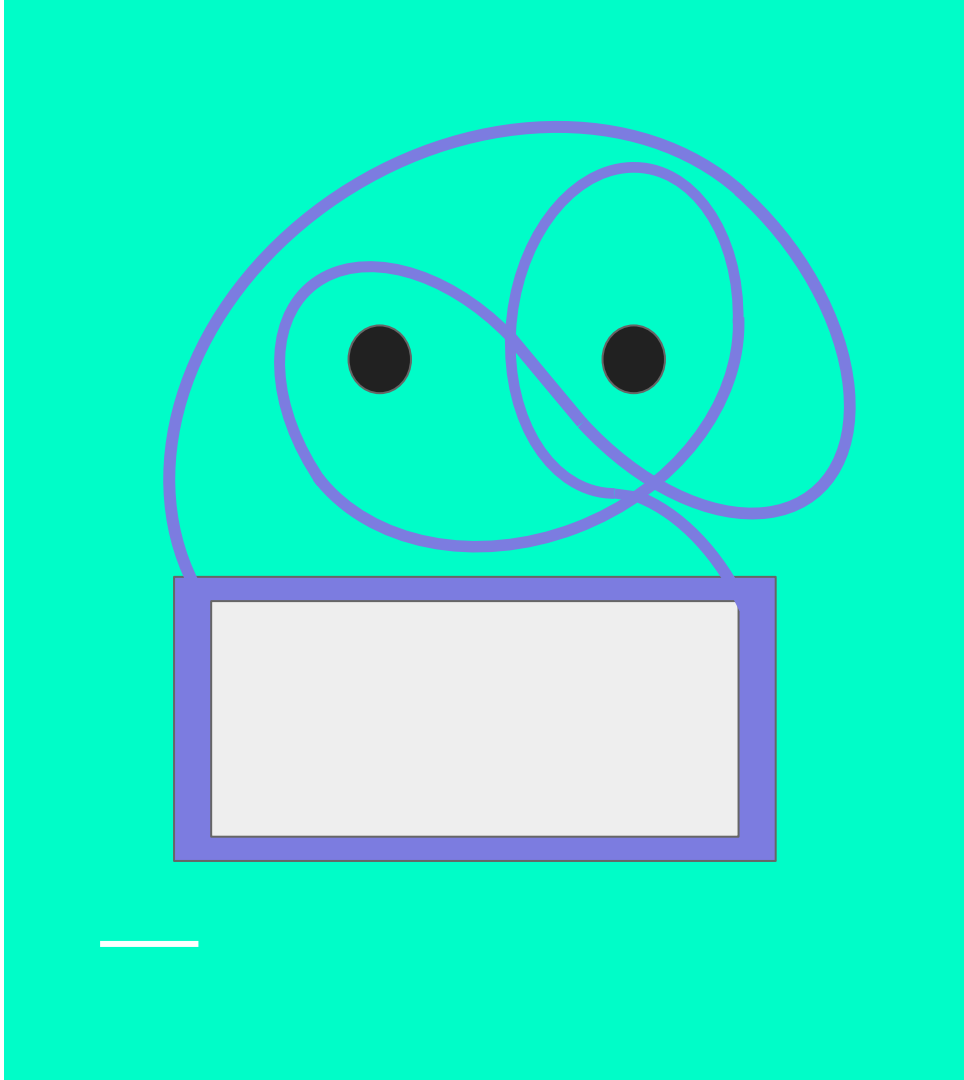

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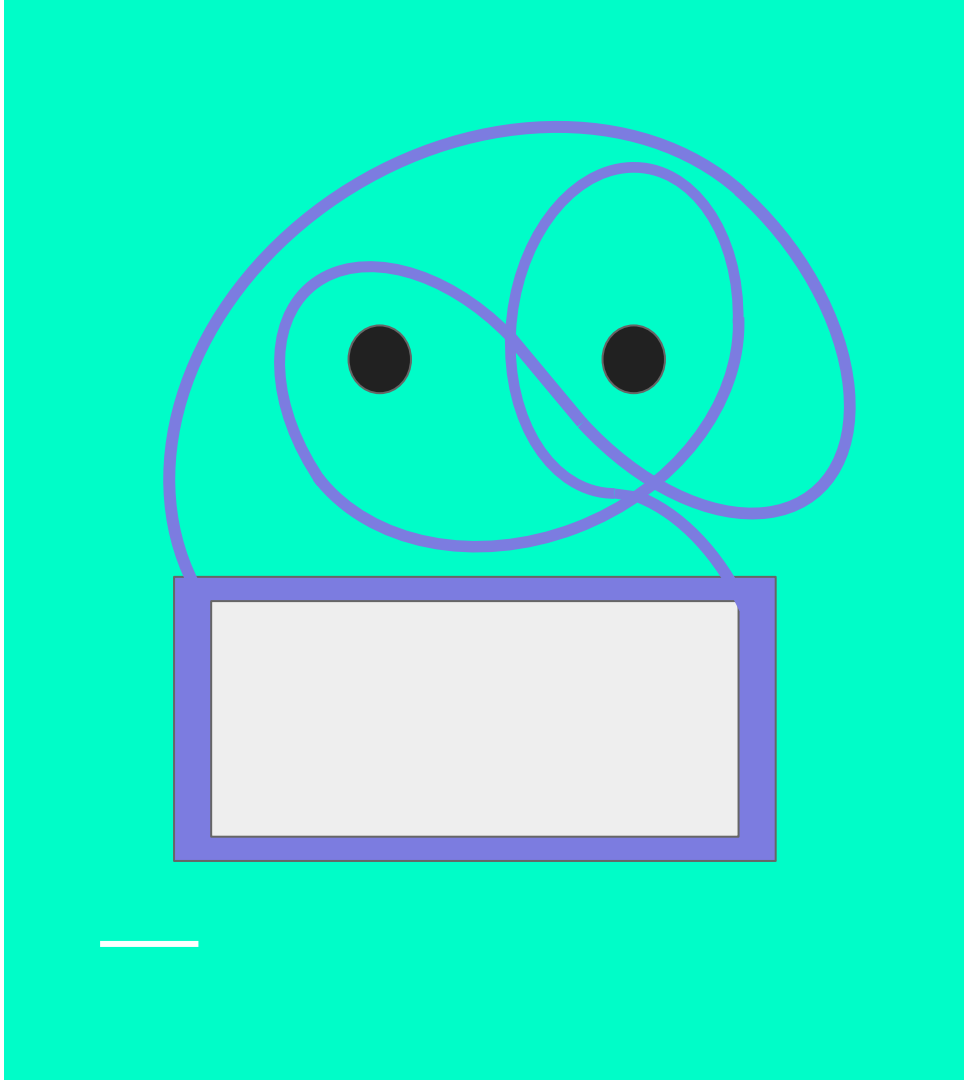
COMMUTATORS



Commutators

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Check out: Chai and Why?(Dec 20, 2020) by Ramprasad Saptharishi

“Commutators! Hanging pictures and solving Rubik’s Cube”

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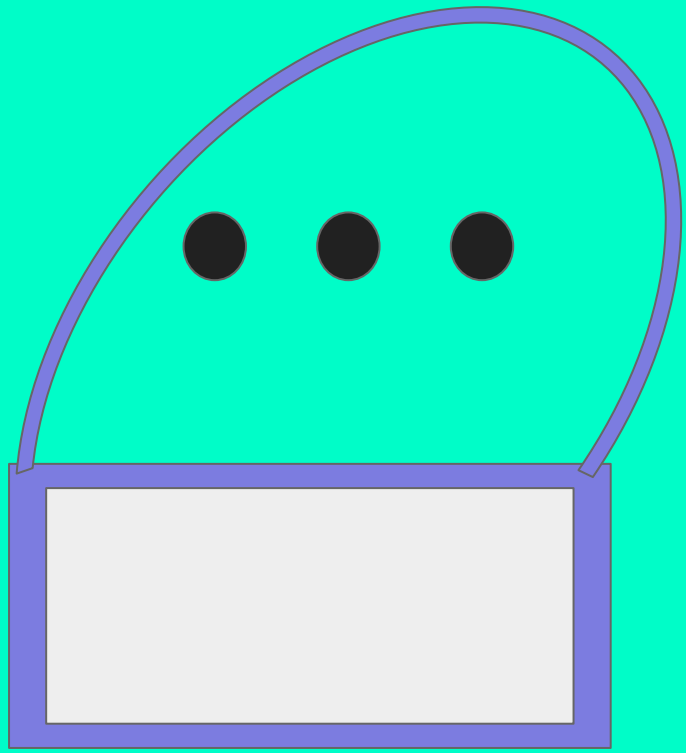

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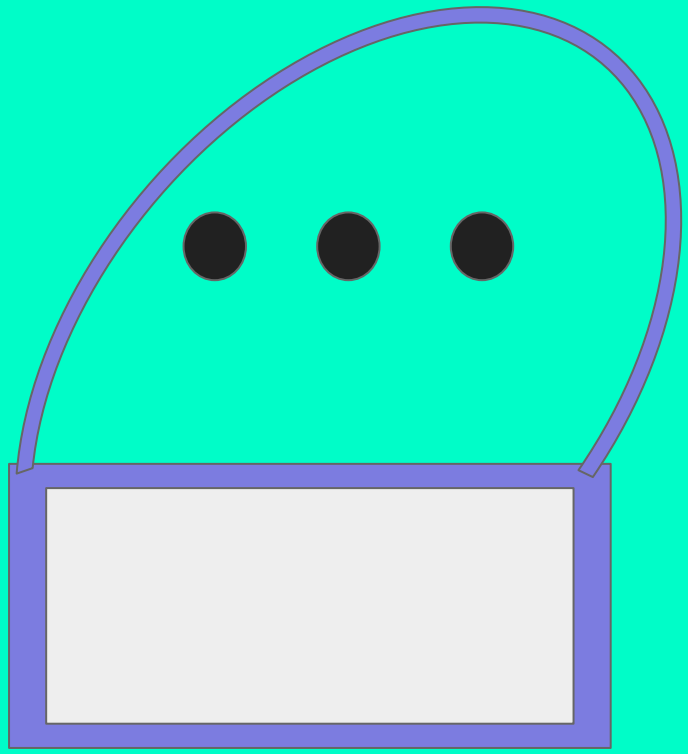
Solution

$$A(BCB^{-1}C^{-1})A^{-1}(CBC^{-1}B^{-1})$$



SCoS @ NISER

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Secure Multiparty Computation

Secure multiparty computation (MPC) is a cryptographic protocol that **allows multiple parties to jointly compute a function** of their inputs while revealing as little as possible about those inputs.

Some of the applications of MPC are **online auctions, voting** etc.

Machine Learning

Machine Learning (ML) is a type of artificial intelligence (AI) that allows software applications to become more accurate in **predicting outcomes without being explicitly programmed to do so**.

Application of Machine learning includes **healthcare, natural language processing, recommendation systems.**

Data Clustering

Data clustering is the process of **grouping data points together based on their similarities**. Clustering is an unsupervised learning technique.

Application of data clustering includes **market segmentation, image segmentation, anomaly detection** and many more.

Algorithm Design

Algorithm Design refers to developing efficient algorithms to **solve computational problems and analysing their complexity**.

The aim is to devise algorithms that optimize time and space complexities, addressing challenges in various domains like **optimization, cryptography, artificial intelligence**, and more.

Complexity Theory

Complexity Theory is the study of different **computational models**.

Research in this area focuses on understanding the **power and limitations** of objects that model **real-world machines** with varied restrictions. Knowing the limitations of a model helps us devise **secure protocols** against them.

THANK YOU !

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