

Assignment 0

Due: 10am on 05/08/24

Instructions

- Discussion is allowed and in fact encouraged
- Answers must be written by yourself.
- All sources that are used to reach the solution must be mentioned.

1. Find the error in the following proof that all horses are the same colour.

CLAIM: In any set of n horses, all horses are the same colour.

Proof: By induction on n .

Base Case: $n = 1$.

In any set containing just one horse, all horses are clearly of the same colour.

Induction Step: For $k \geq 1$, assuming the claim is true for $n = k$ and proving for $n = k + 1$

Take any set H of $k+1$ horses. We will show that all the horses in this set are of the same colour.

Remove one horse from this set to obtain the set H_1 , with just k horses. By the induction hypothesis, all the horses in H_1 are of the same colour. Now replace the removed horse and remove a different horse to obtain the set H_2 . By the same argument, all the horses in H_2 are the same colour. Therefore all the horses in H must be the same colour.

[4]

2. Let A be the set $\{x, y, z\}$ and B be the set $\{x, y\}$.

- (i) Is A a subset of B ?
- (ii) Is B a subset of A ?
- (iii) What is $A \cup B$?

(iv) What is $A \cap B$?

(v) What is $A \times B$?

(vi) What is the power set of B ?

$$[1+1+1+1+1+1]$$

3. Let A, B, C be sets with a, b, c many elements respectively.

i) How many elements does $A \times B$ have?

ii) How many elements does the power set of C have?

$$[2+2]$$

4. Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{6, 7, 8, 9, 10\}$.

The functions $f: X \rightarrow Y$ and $g: X \times Y \rightarrow Y$ are described as follows:

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

g	6	7	8	9	10
1	10	10	10	10	10
2	7	8	9	10	6
3	7	7	8	8	9
4	9	8	7	6	10
5	6	6	6	6	6

- i) What is the value of $g(4, f(4))$?
- ii) What is the range and domain of f ?
- iii) What is the range and domain of g ?

[1+1+1]

5. Consider the undirected graph $G = (V, E)$

where V , the set of nodes, is $\{1, 2, 3, 4\}$
and E , the set of edges, is $\{\{1, 2\}, \{2, 3\},$
 $\{1, 3\}, \{2, 4\}, \{1, 4\}\}$.

- i) Draw the graph G .
- ii) What are the degrees of each node?
- iii) Indicate a path from node 3 to node 4 on your drawing of G .

[$\frac{1}{2} + 2 + \frac{1}{2}$]