Revision: Weeks 4, 5 & 6 Due: 5pm on Sept. 16

Instructions

- Discussion is allowed and infact encouraged
- Answers **must** be written by yourself.
- All sources (including discussions) that are used to reach the solution must be mentioned.

Argue Correctness ≡ Describe what each of the states are supposed to be doing and why they do that.

① Write an algorithm to decide the following language. Construct Turing Machines for each step. Remember to label the states that are common across steps consistently.

$A = \{w : w \text{ contains twice as many } 0\text{s as } 1\text{'s}\}$.

[4+6].

② Explain why the following is not a legitimate decider to check if a given polynomial has integer roots.

M : On input <p>, a polynomial over variables $x_1, \ldots, x_k$,

1. Try all possible settings of $x_1, x_2, \ldots, x_k$ to integer values.
2. Evaluate p on all of these settings
3. If any of these settings evaluate of 0 then accept;
   Otherwise reject.

[2]

(3) Give a formal definition of an enumerator over the alphabet Σ.

[4].

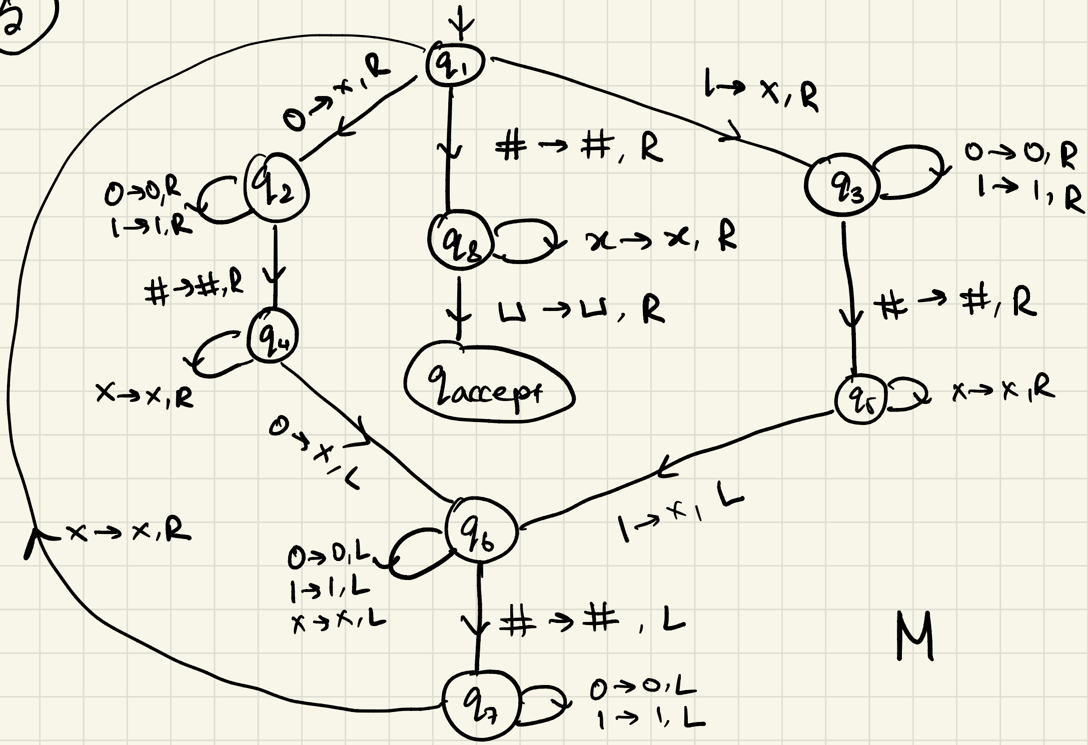
(4) Write an algorithm to decide the following language. (No need to give formal constructions of the TM):

$$ALL_{DFA} = \{ \langle A \rangle : A \text{ is a DFA and } L(A) = \Sigma^* \}$$

[Hint: Use one of the deciders constructed in class as a sub-routine]

[4]

(5)



M

Give the sequence of configurations that M enters on the following input:

i) 1 ## 1

ii) 10 # 11

[2+ 2]

(6) If L_1, L_2 are recursively enumerable languages, show that $L_1 \cap L_2$ is recursively enumerable as well.

[3]

(7) Show that $\{(i, j, k) : i, j, k \in \mathbb{N}\}$ is countable.

[3]
