

Revision: Week 10

Due: 5pm on Oct 30th

Instructions

- Discussion is allowed and in fact encouraged
- Answers **must** be written by yourself.
- All sources (including discussions) that are used to reach the solution must be mentioned.

① If $A \leq_m B$ and B is a regular language, does that imply that A is a regular language? Why or why not? [4].

② Show that \leq_m is a transitive relation. [3].

③ Show that A is Turing recognisable iff $A \leq_m \text{ATM}$ [4]

④ Show that both conditions in Rice's

theorem are necessary.

That is,

(i) Give an example of a trivial property and show that languages with this property are decidable.

(ii) Give an example of a non-trivial property, which is decidable, s.t.

$\exists M_1, M_2$ for which $L(M_1) = L(M_2)$

but $\langle M_1 \rangle \in P$ & $\langle M_2 \rangle \notin P$.

↳ P is the property [2+3]

⑤ Answer True or False, giving reasons for your answer.

(i) $3^n = 2^{O(n)}$

(ii) $1 = o(1/n)$ [2+2]