Revision : Week 11

Due : 5 pm on Oct 6^{th}.

Instructions : The usual ones ... ☺

① Let CONNECTED = $\{\langle G \rangle$: G is a connected undirected graph$\}$

Analyse the following algorithm to show that this language is in P.

M: On input $\langle G \rangle$, the encoding of a graph G,

1. Select the first node of G and mark it.
2. Repeat the following until no new nodes are marked
   - i) For each node in G, mark it if it is attached by an edge to a node that is already marked.
3. Scan all the nodes of G to determine whether they all are marked.
   - If they are, accept
   - Otherwise, reject.

[4]

(2) Note that any integer has a binary representation. If $m = \sum_{i=0}^{k} 2^i b_i$

then $(b_k b_{k-1} \cdots b_0)$ is called the binary representation of m and k is said to be its length.

a) What is the length of the binary representation of m for a given $m \in \mathbb{N}$?

b) For a given $m \in \mathbb{N}$, gives an $O(\sqrt{m})$ algorithm that checks if m is prime.

c) Why does (b) not prove that the following language is in P?

$$\text{PRIME} = \{ n : n \in \mathbb{N} \text{ is a prime number} \}.$$

d) Show that for any $m \in \mathbb{N}$,

$$\text{SUM}_m = \{ (a,b) : a, b \in \mathbb{N} \ \& \ a+b = m \} \in P.$$

[1+2+2+5].

(3) Call graphs G and H isomorphic if the nodes of G may be reordered so that it is identical to H.

Let $ISO = \{ \langle G, H \rangle : G \& H$ are isomorphic graphs $\}$.

Show that $ISO \in NP$. [3]

(4) Is $f : \{0,1\}^2 \to \{0,1\}$ defined by

$$f(x,y) = (x \vee y) \wedge (x \vee \bar{y}) \wedge (\bar{x} \vee y) \wedge (\bar{x} \vee \bar{y})$$

satisfiable? Why? [1+2]

(5) a) Show that P is closed under complement.

b) Would a similar statement hold for NP?

That is,

i) How would you define the set of all languages whose complement is in NP?

ii) Do you think this class is in NP? Why?

[4+4+2]