

Revision: Week 13

Due on Nov 18, 2024.

Instructions: The usual ones... 😊

① Prove the two claims from the notes of Lecture 27. [4+4]

② Define $\text{coNP} = \{ L : \text{the complement of } L \text{ is in NP} \}$.

Why is the following not a correct proof of $\text{coNP} \subseteq \text{NP}$?

Let $L \in \text{coNP}$. Then there is an efficient non-deterministic TM N that decides the complement of L . Define another non-deterministic TM as follows:

N' : On input w ,

1. Run N on w .

2. If it accepts, reject and if it rejects, accept.

Since N is efficient, so is N' .

Therefore $L \in \text{coNP}$.

[3]

(3) A subset of nodes of a graph G is a dominating set if every other node of G is adjacent to some node in the subset.

Let

$$\text{DOMINATING-SET} = \left\{ \langle G, k \rangle \mid G \text{ has a dominating set of size } k. \right\}$$

Show that this is NP-Complete. [4]

[Hint: Recall that VERTEX-COVER is NP-Complete]

(4) Given a CNF formula ϕ with n variables and c clauses, construct an NFA with $O(cn)$ states that accepts all non-satisfying assignments represented as Boolean strings of length n [5].