Problem Set : Week 5

Due: 2pm on 12/9/25

Instructions

- Discussion is allowed and infact encouraged
- Answers must be written by yourself.
- All sources that are used to reach the solution must be mentioned.

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(1) Given an $m \times n$ matrix T over $\mathbb{R}$, show that $(S, \mathcal{I})$ is a matroid where S is the set of columns of T and $\forall A \subseteq T$

$A \in \mathcal{I} \iff$ columns of A are lin. indep.

> You can't just state some math lemmas Please prove it yourself.

[4]

(2) Let $(S, \mathcal{I})$ be a matroid. Show that the maximal independent sets in $\mathcal{I}$ have the same size. Also show that $(M, \mathcal{I}')$ is a matroid, where $\mathcal{I}' = \{ A' : S \setminus A' \text{ contains some maximal } A \in \mathcal{I} \}$. [1+5]

(3) If an algorithm were performing n operations that included:

- Push (S, x)
- Multipush $(S, x_1, \ldots, x_k)$
- Pop (S)
- Multipop (S, k)

then would the amortised cost of each operation still be $O(1)$? [2]

You can assume that the stack is implemented using an array with m cells. (m: some arb. fixed const.)

(4) Show that if a 'Decrement' operation was included along with the 'Increment' operation, in a k-bit counter, then n operations could cost as much as $\Theta(nk)$. [3]

(k: some arb. fixed const.)

(5) Suppose we perform a sequence of n operations on a data structure in which the i^{th} operation costs

$$\begin{cases} i & \text{if } i \text{ is an exact power of } 2 \\ 1 & \text{otherwise.} \end{cases}$$

What is the amortised cost per operation? [3]

(6) Illustrate the result after each of the following operations performed in sequence on an initially empty stack stored in array S[1, ..., 6].

Push (S, 4)
Push (S, 1)
Push (S, 3)
Pop (S)
Push (S, 8)
Pop (S).

So the illustration should contain 6 stacks in total showing the state of the stack after each of these steps.

[2]
