Problem Set : Week 7. Due: 2pm on 24/9/25

Instructions

- Discussion is allowed and infact encouraged
- Answers must be written by yourself.
- All sources that are used to reach the solution must be mentioned.

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① Suppose you are given a bipartite graph G and you want to use ChatGPT to help you check whether it has a perfect matching or not.

Suppose you ask it.

"Does G have a perfect matching?"

and it answers YES. To be sure that it isn't lying, you can then ask

"Give a perfect matching".

You can now check if whatever it gives is indeed a perfect matching or not and convince yourself that it is not lying.

On the other hand, suppose it answered 'NO' to "Does G have a perfect matching?"

Again you would like to check if it lying or not. What question would you ask in this case to ChatGPT so that you can use the 'proof' it provides to be convinced that it is telling the truth? [2]

(2) Let $\{u, v\}$ be a minimum weight edge in a connected graph G. Show that it must belong to some minimum spanning tree. [3].