Problem Set : Week 8. Due: 5pm on 22/10/25

Instructions

- Discussion is allowed and infact encouraged
- Answers must be written by yousef.
- All sources that are used to reach the solution must be mentioned.

① Write a pseudocode that executes the following "greedy" strategy

Define the density of a rod of length i to be P/i, that is, its value per inch. The strategy for a rod of length n is to cut off a first piece of length i (for $1 \leq i \leq n$) which has maximum density. Continue by applying this strategy to the remaining piece of length $n-i$.

Show by means of a counterexample, that this does not always give the optimal solution. [4+2]

(2) The fibonacci sequence is described as follows: $f_0 = 1$, $f_1 = 1$

and $\forall\, n \geqslant 2$ $f_n = f_{n-1} + f_{n-2}$.

i) Give an $O(n)$-time algorithm to compute the n-th Fibonacci number.

ii) Draw the sub-problem graph. How many vertices and edges are in the graph? [2+3]

(3) Explain why dynamic programming does not help speed up the MERGE-SORT algorithm. [2]

(4) Give an $O(n^2)$-time algorithm to find the longest monotonically increasing subsequence of a sequence of n numbers. [4]

(5) Suppose that in the rod-cutting problem, there is also a limit l_i on the number of pieces of length i that we are allowed to produce $(1 \leq i \leq n)$. Show that the optimal sub-structure property no longer holds. [3].