

Quiz 1.

- ① Draw a boolean circuit for:

$$\text{XOR} : \{0,1\}^4 \rightarrow \{0,1\}$$

[3]

- ② Recall that $\text{SIZE}(\text{ADD}_n) = O(n)$. Use this to show that $\sum_{i=0}^{n-1} \text{SIZE}(\text{ADD}_{n+i}) = O(n^2)$. [5]

- ③ Consider the threshold function:

Fun Fact: Neural Networks are essentially circuits in which each gate is a threshold gate.

$$\text{Thr}_{a_0, a_1, a_2, b} : \{0,1\}^3 \rightarrow \{0,1\} \quad \text{where}$$

$$\text{Thr}_{a_0, a_1, a_2, b}(\vec{x}) = \begin{cases} 1 & \text{if } \sum_{i=0}^2 a_i x_i > b \\ 0 & \text{otherwise} \end{cases}$$

$$\text{with } a_0, a_1, a_2, b \in \{-8, -7, \dots, -1, 0, 1, \dots, 7, 8\}.$$

- i) What is $\text{Thr}_{1,1,1,2}(x_0, x_1, x_2)$? [1]

- ii) Choose any valid (a_0, a_1, a_2, b) apart from $(1,1,1,2)$ or $(0,0,0,*)$. [1]

- iii) Draw the truth table for $\text{Thr}_{a_0, a_1, a_2, b}$ where (a_0, a_1, a_2, b) is what you chose in (ii). [8]