

Quiz 4.

① Show that $\mathbb{P} \subseteq \text{EXP}$ where

$$\mathbb{P} = \bigcup_{k \in \mathbb{N}} \text{TIME}(n^k) \text{ and}$$

$$\text{EXP} = \bigcup_{k \in \mathbb{N}} \text{TIME}(2^{n^k}) \quad [2]$$

② Define $\text{IntRoots} : \mathbb{F}[x] \rightarrow \{0, 1\}$ by

$$\text{IntRoots}(f) = \begin{cases} 1 & \text{if } f \text{ has an integer root.} \\ 0 & \text{o.w.} \end{cases}$$

Why does the following TM not compute IntRoots ?

M: On input $\langle p \rangle$, a polynomial over variables $x_1, \dots, x_k$,

1. for $(a_1, \dots, a_n) \in \mathbb{Z}^n$

2. $\begin{cases} \text{if } f(\vec{a}) = 0 \\ \quad \text{return 1} \end{cases}$

3. return 0.

[2]

③ a) Given a TM M and an input x , describe a TM $M_{M,x}$ that halts on every input iff M halts on x . [6]

b) Let D be the set of all TMs that halts on every input. Use (a) to show that $f: \{0,1\}^* \rightarrow \{0,1\}$ defined by

$$f(z) = \begin{cases} 1 & \text{if } z \in D \\ 0 & \text{o.w.} \end{cases}$$

is not computable. [4]

④ Define $E_{TM}: \{0,1\}^* \times \{0,1\}^* \rightarrow \{0,1\}$
by $E_{TM}(M, x) = \begin{cases} 1 & \text{if } M(x) = 0 \ \forall x \\ 0 & \text{o.w.} \end{cases}$

and $EQ_{TM}: \{0,1\}^* \times \{0,1\}^* \rightarrow \{0,1\}$

by $EQ_{TM}(M, M') = \begin{cases} 1 & \text{if } M(x) = M'(x) \\ 0 & \text{o.w.} \end{cases} \quad \underline{\forall x}$

Show that $E_{TM} \leq EQ_{TM}$. [6]