Quiz 6

① Suppose $L_1, L_2 \in NP$. Show that $L_1 \cup L_2 \in NP$.

[Hint: Use the NTM characterisation]   [4]

② Show that the relation $\leq_p$ among boolean functions is

i) Reflexive $(f \leq_p f \;\; \forall f)$ [1]

ii) Transitive $(f \leq_p g \; \& \; g \leq_p h \Rightarrow f \leq_p h)$ [2]

iii) Not symmetric   [3]

③ Suppose φ is a boolean formula in CNF.

i) Show that $\neg \varphi$ is a DNF of the
                                    same size. [2]

ii) What can you say about the satisfiability
   of $\neg \varphi$ if   a) φ is satisfiable?   [1]
                          b) φ is unsatisfiable? [1]

④ Define $UNSAT = \{\varphi : \varphi \text{ is an unsatisfiable CNF}\}$

i) Show that $\{\varphi : \varphi \text{ is a CNF}\} \setminus UNSAT \in NP$.

ii) Do you think $UNSAT \in NP$?

iii) Show that $\overline{SAT} \leq_P^T SAT$.

iv) Do you think $\overline{SAT} \leq_P SAT$?

$$[1 + 2 + 1 + 2]$$